Intercepts, stationary behaviour and asymptotes

6 questions | 26 marks

Equivalent correct methods earn the corresponding marks.

1. For $f(x)=\frac{x^2}{x-1}$, find asymptotes and stationary points. [5 marks]

Division gives $x+1+\frac{1}{x-1}$.

$f'(x)=\frac{x(x-2)}{(x-1)^2}$.

The derivative changes from positive to negative at 0 and negative to positive at 2.

Answer: Asymptotes $x=1$, $y=x+1$; local maximum $(0,0)$, local minimum $(2,4)$.

2. For $f(x)=\frac{x^2-4}{x^2-1}$, give intercepts, asymptotes and its stationary point. [5 marks]

Rewrite as $1-\frac{3}{x^2-1}$.

$f'(x)=\frac{6x}{(x^2-1)^2}$, changing from negative to positive at 0.

Answer: Roots ± 2; y-intercept 4; vertical asymptotes ± 1; horizontal asymptote 1; local minimum $(0,4)$.

3. Find the stationary points of $f(x)=x+\frac{1}{x}$ and classify them. [4 marks]

$f'(x)=1-\frac{1}{x^2}$.

The critical inputs are -1 and 1, with $x=0$ outside the domain.

Use derivative signs on the separate branches.

Answer: Local maximum $(-1,-2)$ and local minimum $(1,2)$.

4. For $f(x)=\frac{x^3}{x^2-1}$, explain why the origin is not a local extremum. [4 marks]

$f'(x)=\frac{x^2(x^2-3)}{(x^2-1)^2}$.

Its sign does not change at 0; the odd graph has a stationary inflection there.

Answer: The derivative vanishes at 0 but is negative immediately on both sides.

5. Find all x-intercepts of $f(x)=\frac{x+1}{x^2-1}$ and explain any excluded apparent intercept. [3 marks]

The numerator vanishes only at $x=-1$, which is excluded by the denominator.

The simplified reciprocal rule has no zero.

Answer: There are no x-intercepts.

6. Find the range of $f(x)=\frac{x^2}{x-1}$ using its extrema or a discriminant. [5 marks]

$y(x-1)=x^2$ gives $x^2-yx+y=0$.

Real x requires $y^2-4y\geq 0$, so y is at most 0 or at least 4.

The excluded input $x=1$ is never a root of this equation.

Answer: $(-\infty,0]\cup[4,\infty)$.