Rational functions

6 questions | 20 marks

Equivalent correct methods earn the corresponding marks.

1. Find the hole in $f(x)=\frac{x^2-1}{x-1}$. [3 marks]

For x not equal to 1, simplify to $x+1$.

The excluded original input remains excluded.

Answer: $(1, 2)$.

2. Divide $\frac{x^2+1}{x-1}$ and state the oblique asymptote. [3 marks]

Polynomial division gives quotient $x+1$ and remainder 2.

The remainder term tends to zero as the magnitude of x grows.

Answer: $x+1+\frac{2}{x-1}$; asymptote $y=x+1$.

3. Analyse $f(x)=\frac{2x+1}{x-3}$: give asymptotes and intercepts. [4 marks]

Rewrite as $2+\frac{7}{x-3}$.

Set the numerator to zero for the x-intercept; substitute $x=0$ for the other.

Answer: Asymptotes $x=3$, $y=2$; intercepts $(-\frac{1}{2}, 0)$ and $(0, -\frac{1}{3})$.

4. Find the stationary points and range of $f(x)=\frac{x}{x^2+1}$. [4 marks]

$f'(x)=\frac{1-x^2}{(x^2+1)^2}$.

The sign changes at -1 and 1 establish the extrema.

Answer: Minimum $(-1, -\frac{1}{2})$, maximum $(1, \frac{1}{2})$; range $[-\frac{1}{2}, \frac{1}{2}]$.

5. For $f(x)=\frac{x-2}{x^2-4}$, state the domain, simplified rule and discontinuities. [4 marks]

Factor the denominator into $(x-2)(x+2)$.

Cancellation does not restore the excluded input 2.

Answer: Domain excludes ± 2; $f(x)=\frac{1}{x+2}$ for allowed x; pole at -2 and hole at $(2, \frac{1}{4})$.

6. Can a graph cross a horizontal asymptote? Use $f(x)=\frac{x}{x^2+1}$ to justify your answer. [2 marks]

A horizontal asymptote describes end behaviour, not a forbidden y-value.

Answer: Yes; it crosses its horizontal asymptote $y=0$ at the origin.