Reciprocal/composite transformations

6 questions | 19 marks

Equivalent correct methods earn the corresponding marks.

1. For $f(x) = x^2 - 1$, identify vertical asymptotes, intercepts and the stationary point of $\frac{1}{f(x)}$. [4 marks]

$g(x) = \frac{1}{x^2 - 1}$.

$g'(x) = -\frac{2x}{(x^2 - 1)^2}$, positive before 0 and negative after 0 within the central branch.

Answer: Vertical asymptotes $x = \pm 1$; y-intercept and local maximum $(0, -1)$; no x-intercepts.

2. Find the range and asymptote of $y = \frac{1}{x^2 + 1}$. [3 marks]

The denominator is at least 1 and grows without bound.

The maximum 1 occurs at $x = 0$; zero is approached but never reached.

Answer: Range $(0, 1]$; horizontal asymptote $y = 0$.

3. Which points on a graph $y = f(x)$ remain fixed when changing to $y = \frac{1}{f(x)}$? [3 marks]

Solve $y = \frac{1}{y}$, with y nonzero.

This gives $y^2 = 1$.

Answer: Points where $f(x) = 1$ or $f(x) = -1$.

4. For $g(x) = \frac{2}{x-1} + 3$, state its asymptotes, intercepts and centre of symmetry. [4 marks]

For $y = 0$, solve $2 + 3(x - 1) = 0$.

Substitute $x = 0$ for the y-intercept.

Answer: $x = 1$, $y = 3$; x-intercept $\frac{1}{3}$, y-intercept 1; centre $(1, 3)$.

5. Find the domain of $y = \frac{1}{\sqrt{x}}$ and explain the endpoint. [3 marks]

The square root needs x nonnegative.

The reciprocal also excludes $x = 0$.

Answer: $x > 0$.

6. If $f(a) = -4$, find the corresponding ordinate on $y = \frac{1}{f(x)}$. Does its sign change? [2 marks]

Reciprocation preserves the sign of a nonzero real number.

Answer: $-\frac{1}{4}$; the sign stays negative.