Inverse functions and restricted domains

6 questions | 20 marks

Equivalent correct methods earn the corresponding marks.

1. Find the inverse of $f(x) = \frac{2x+1}{3}$. [3 marks]

Write $y = \frac{2x+1}{3}$.

Solve for x: $x = \frac{3y-1}{2}$, then swap the variable names.

Answer: $f^{-1}(x) = \frac{3x-1}{2}$.

2. Find the inverse of $f(x) = (x-3)^2 + 2$, restricted to $x \geq 3$. Give its domain and range. [4 marks]

$y - 2 = (x-3)^2$.

The chosen branch has $x - 3$ nonnegative, so take the positive square root.

Swap the original domain and range.

Answer: $f^{-1}(x) = 3 + \sqrt{x-2}$; domain $[2, \infty)$, range $[3, \infty)$.

3. Find the inverse of $f(x) = (x-3)^2 + 2$ when $x \leq 3$. [3 marks]

The original branch has $x - 3$ nonpositive.

Choose the negative square-root branch before swapping variables.

Answer: $f^{-1}(x) = 3 - \sqrt{x-2}$, $x \geq 2$.

4. Find the inverse of $f(x) = \frac{x}{x-1}$, $x \neq 1$. [4 marks]

$y(x-1) = x$ gives $x(y-1) = y$.

The function is its own inverse on the stated domain.

Answer: $f^{-1}(x) = \frac{x}{x-1}$, $x \neq 1$.

5. Find the inverse of $f(x) = 3e^x - 4$ and its domain. [3 marks]

Solve $y + 4 = 3e^x$ for x.

The original range is greater than -4.

Answer: $\ln(\frac{x+4}{3})$, $x > -4$.

6. Find a domain restriction for $f(x) = |x+1|$ that gives inverse $x - 1$, and state the inverse domain. [3 marks]

On the right branch, $f(x) = x + 1$.

The branch range is nonnegative.

Answer: Restrict f to $x \geq -1$; then $f^{-1}(x) = x - 1$ for $x \geq 0$.