One-to-one functions

6 questions | 16 marks

Equivalent correct methods earn the corresponding marks.

1. Is $f(x)=x^2$ one-to-one on the real numbers? Justify your answer. [2 marks]

Use $f(2)=f(-2)=4$ even though the inputs differ.

Answer: No.

2. Show $f(x)=3x-7$ is one-to-one on the real numbers. [3 marks]

If $3a-7=3b-7$, then $3a=3b$ and $a=b$.

Answer: It is one-to-one.

3. Is $f(x)=\frac{1}{x}$, $x\neq 0$, one-to-one? [3 marks]

If $\frac{1}{a}=\frac{1}{b}$ for nonzero a, b, multiplying by ab gives $a=b$.

Its separated branches do not share any y-value.

Answer: Yes.

4. Find two distinct inputs with the same output for $f(x)=x^3-3x$. [3 marks]

$x^3-3x=x(x^2-3)$.

Use two different roots.

Answer: 0 and $\sqrt{3}$ both give 0.

5. Give two maximal intervals on which $f(x)=(x-2)^2$ is one-to-one. [3 marks]

The graph is strictly decreasing on the first interval and increasing on the second.

Each restriction passes the horizontal-line test.

Answer: $(-\infty, 2]$ and $[2, \infty)$.

6. Explain why $f(x)=2^x$ has at most one input for each output. [2 marks]

For $a<b$, raising a base greater than 1 to these powers gives $2^a<2^b$.

A horizontal line meets its graph at most once.

Answer: It is strictly increasing.