Composition and domain restrictions

6 questions | 20 marks

Equivalent correct methods earn the corresponding marks.

1. Let $f(x)=\sqrt{x-1}$ and $g(x)=2x+3$. Find both compositions and their domains. [4 marks]

For f after g, require $2x+3\geq 1$.

For g after f, the inner root requires $x\geq 1$; g accepts every real output.

Answer: $f(g(x))=\sqrt{2x+2}$, $x\geq -1$; $g(f(x))=2\sqrt{x-1}+3$, $x\geq 1$.

2. Let $f(x)=\frac{1}{x-2}$ and $g(x)=x^2$. Find the domain of $f\circ g$. [3 marks]

$f(g(x))=\frac{1}{x^2-2}$.

Require $x^2-2\neq 0$.

Answer: All real x except $\pm\sqrt{2}$.

3. For $f(x)=\ln x$ and $g(x)=3-x$, find $(f\circ g)(x)$ and its domain. [3 marks]

The logarithm requires $3-x>0$.

Answer: $\ln(3-x)$, $x<3$.

4. Let $f(x)=\sqrt{x}$ and $g(x)=x^2-4$. Find the domain of $g\circ f$. [3 marks]

The inner square root must be defined.

After substitution, $g(f(x))=x-4$, but the original inner-domain restriction remains.

Answer: $x\geq 0$.

5. Let $f(x)=x+1$ and $g(x)=\frac{1}{x}$. Compare $f\circ g$ and $g\circ f$. [4 marks]

Substitute into the outer rule in each order.

The expressions and excluded inputs differ.

Answer: $\frac{1}{x}+1$ with $x\neq 0$; $\frac{1}{x+1}$ with $x\neq -1$.

6. Let $f(x)=\sqrt{4-x^2}$ and $g(x)=x-1$. Find the domain of $f\circ g$. [3 marks]

Require $-2\leq x-1\leq 2$.

Add 1 throughout.

Answer: $-1\leq x\leq 3$.