Complex Numbers - Paper 5

Name: _____ Date: _____

10 questions | 56 marks | 70 minutes

Applications and geometry: transformations, distance constraints, phase relations and coefficient models.

A scientific calculator may be used.

1. A planar motion uses $T(z) = (2 + i)z$. [5 marks]

(a) Find $T(1 - i)$ in Cartesian form. [2 marks]

(b) State the scale factor and give the rotation angle exactly using an inverse tangent. [3 marks]

2. A complex signal filter acts by $w = \frac{z}{1+i}$. The input is $z = 6i$. [5 marks]

(a) Find the output w. [2 marks]

(b) Compare output amplitude and phase with input amplitude and phase. [3 marks]

3. A point z is moved by $w = iz + 2 - i$. [6 marks]

(a) Find the image of $z = 1 + 2i$. [2 marks]

(b) Find the fixed point, where $w = z$. [4 marks]

4. A circular tracking zone has equation $|z - (2 + i)| = \sqrt{5}$. [6 marks]

(a) Find every intersection with the real axis. [3 marks]

(b) Find the greatest distance from the origin of any point in the closed zone $|z - (2 + i)| \leq \sqrt{5}$. [3 marks]

5. Two beacons represent $a = -1 + i$ and $b = 3 + i$. A receiver is equally distant from both. [6 marks]

(a) Find the receiver's possible locus. [3 marks]

(b) Find its possible positions if it is also 2 units from i. [3 marks]

6. Consider $\text{Arg}(z - (1 + i)) = -\frac{\pi}{4}$. [5 marks]

(a) Give a parametric description using one positive real parameter t. [3 marks]

(b) Find its intersection with the real axis. [2 marks]

7. The map $w = (1 + i)z$ acts on the circle $|z - 2| = 1$. [5 marks]

(a) State the centre and radius of the image circle. [3 marks]

(b) Write a modulus equation for the image circle. [2 marks]

8. A repeated steering instruction multiplies a position by $q = \text{cis}(\frac{2\pi}{9})$. [6 marks]

(a) Find the least positive number of instructions returning every position to its original value. [3 marks]

(b) The instruction instead uses $2q$. Can a nonzero point ever return exactly after a positive number of repetitions? [3 marks]

9. A real monic polynomial has degree 4 and roots $2+i$ and $2+3i$. [6 marks]

(a) Give its factorisation into real quadratics. [3 marks]

(b) Find its constant term and coefficient of z^3. [3 marks]

10. An algorithm proposes two solutions of $z^2=5+12i$: $3+2i$ and $3-2i$. [6 marks]

(a) Identify the invalid solution and replace it. [3 marks]

(b) Explain why conjugating one root did not give the other root. [3 marks]