Complex Numbers - Paper 4

Name: _____ Date: _____

10 questions | 59 marks | 70 minutes

Roots and factorisations, from recognising a square to exact root geometry and real coefficient constraints.

A scientific calculator may be used.

1. Let $z = 1 - \sqrt{3}i$. [5 marks]

(a) Write z in principal-argument polar form. [2 marks]

(b) Write its reciprocal in polar and Cartesian forms. [3 marks]

2. A complex square is $3 + 4i$. [5 marks]

(a) Calculate $(2 + i)^2$. [2 marks]

(b) Hence solve $z^2 = 3 + 4i$ and explain why your list is complete. [3 marks]

3. Find the cube roots of $-27i$. [6 marks]

(a) Give all roots with principal arguments. [3 marks]

(b) Convert the roots to Cartesian form and find their sum. [3 marks]

4. Consider $z^5 = -32$. [6 marks]

(a) Give a formula listing all five roots. [3 marks]

(b) Identify the real root and state how many roots have positive real part. [3 marks]

5. The roots of $z^4 = 16i$ are $z_k = 2\text{cis}(\frac{\pi}{8} + \frac{k\pi}{2})$ for $k = 0, 1, 2, 3$. [6 marks]

(a) Show that opposite roots sum to zero and hence find the sum of all roots. [3 marks]

(b) Calculate the product of all four roots in polar and Cartesian forms. [3 marks]

6. Consider $P(z) = z^6 - 1$. [7 marks]

(a) List the six roots in polar form. [3 marks]

(b) Factorise P into real linear and irreducible real quadratic factors. [4 marks]

7. Consider $P(z) = z^4 + 4$. [6 marks]

(a) Show that $P(z) = (z^2 - 2z + 2)(z^2 + 2z + 2)$. [3 marks]

(b) Find every complex root. [3 marks]

8. A monic real cubic has a root $1 + 2i$ and its coefficient of z^2 is -7. [6 marks]

(a) Find all its roots. [3 marks]

(b) Find the cubic in expanded form. [3 marks]

9. The real polynomial $P(z) = (z^2 + 4)^2(z - 1)^2$ has degree 6. [6 marks]

(a) List its distinct roots and their multiplicities. [3 marks]

(b) Explain how the root count agrees with the degree and with conjugate pairing. [3 marks]

10. A designer places five equally spaced markers at the roots of $z^5 = 32$. [6 marks]

(a) Find the distance between neighbouring markers in exact trigonometric form. [3 marks]

(b) An enlargement multiplies all positions by $3\text{cis}(\frac{\pi}{10})$. Give the new radius and the new first marker's argument if the original first marker was 2. [3 marks]