Conjugate-root results for real polynomials

6 questions | 15 marks

Equivalent correct methods earn the corresponding marks.

1. Find a real monic quadratic with roots $3 \pm i$. [2 marks]

$(z-3)^2+1 = z^2-6z+10$.

Answer: $z^2-6z+10$.

2. A monic real quartic has roots i, $2+i$ and their conjugates. Factorise it over $\mathbb{R}$. [2 marks]

The conjugate pairs are $\pm i$ and $2 \pm i$.

Answer: $(z^2+1)(z^2-4z+5)$.

3. A monic real cubic has root $1+i$ and constant term -6. Find the remaining real root. [3 marks]

The conjugate pair gives z^2-2z+2. Write $(z-r)(z^2-2z+2)$; its constant term is $-2r=-6$.

Answer: 3.

4. Explain why a real polynomial of degree 5 cannot have exactly one non-real root counting multiplicity. [3 marks]

One non-real root would leave its forced conjugate uncounted.

Answer: Every non-real root must be accompanied by its distinct conjugate, so the non-real count is even.

5. For real a and b, z^2+az+b has root $1-\sqrt{2}i$. Find a and b. [2 marks]

The pair gives $(z-1)^2+2 = z^2-2z+3$.

Answer: $a=-2$, $b=3$.

6. Test the claim: if i is a root of $P(z)=z-i$, then $-i$ is also a root. [3 marks]

The coefficient $-i$ is non-real, so the conjugate-root theorem does not apply.

Answer: The claim is false: $P(-i)=-2i \neq 0$.