Complex loci and geometry

6 questions | 14 marks

Equivalent correct methods earn the corresponding marks.

1. Give the centre and radius of $|z+2-i|=4$. [2 marks]

Write the expression as $|z-(-2+i)|$.

Answer: Centre $-2+i$; radius 4.

2. Write $|z-(2-i)|=3$ as a Cartesian equation. [2 marks]

The real and imaginary differences are $x-2$ and $y+1$.

Answer: $(x-2)^2+(y+1)^2=9$.

3. Find the locus $|z-i|=|z+i|$. [2 marks]

$x^2+(y-1)^2=x^2+(y+1)^2$ simplifies to $-4y=0$.

Answer: The real axis, $y=0$.

4. Describe $\operatorname{Arg}(z-1)=\frac{\pi}{2}$. [3 marks]

$z-1$ must be a positive imaginary number. The point $z=1$ is excluded.

Answer: The vertical ray $x=1,\ y>0$.

5. Find the points satisfying $|z|=2$ and $\operatorname{Re} z=1$. [3 marks]

$x=1$ in $x^2+y^2=4$ gives $y^2=3$. Both signs satisfy the circle.

Answer: $z=1\pm\sqrt{3}i$.

6. A tracking system reports $|z-(3+2i)|\leq 1$. Interpret its possible positions. [2 marks]

The inequality is a closed disc, rather than only its boundary.

Answer: All points at most 1 unit from $(3,2)$, including the circle boundary.