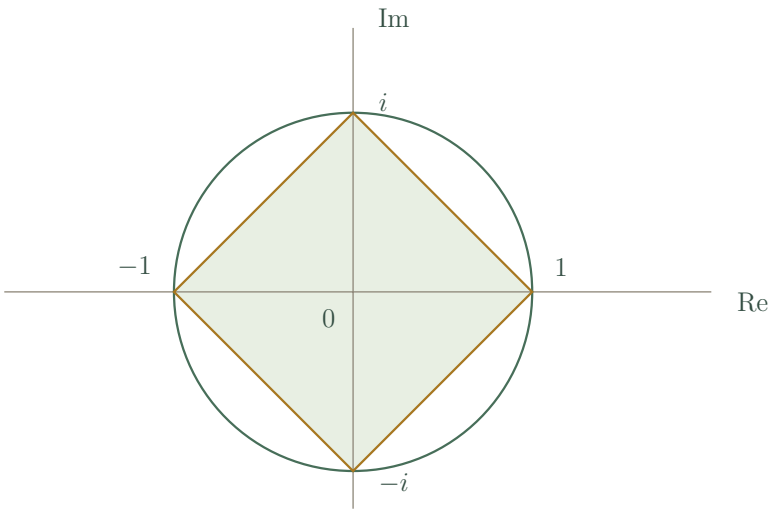
Roots of unity

6 questions | 14 marks

Equivalent correct methods earn the corresponding marks.

1. List all fourth roots of unity in Cartesian form. [2 marks]



The angles are $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$.

Answer: $1, i, -1, -i$.

2. Let $\omega = \text{cis}(\frac{2\pi}{3})$. Simplify ω^{11}. [2 marks]

$11 = 3 \times 3 + 2$, and every factor ω^3 is 1.

Answer: ω^2.

3. For $\omega = \text{cis}(\frac{2\pi}{3})$, show that $\overline{\omega} = \omega^2$. [2 marks]

Conjugation negates the argument; $-\frac{2\pi}{3}$ differs from $\frac{4\pi}{3}$ by a full turn.

Answer: Both equal $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$.

4. Find the product of all fourth roots of unity. [2 marks]

$1 \times i \times (-1) \times (-i) = -1$.

Answer: -1.

5. Show that the sum of the five fifth roots of unity is zero. [3 marks]

Multiply the sum by $1 - \omega$ to get $1 - \omega^5 = 0$. Since $\omega \neq 1$, divide by $1 - \omega$.

Answer: $1 + \omega + \omega^2 + \omega^3 + \omega^4 = 0$, where $\omega = \text{cis}(\frac{2\pi}{5})$.

6. A regular hexagon has vertices at the sixth roots of unity. Find its perimeter. [3 marks]

Adjacent vertices differ by a chord subtending $\frac{\pi}{3}$, of length $2\sin(\frac{\pi}{6}) = 1$. There are six sides.

Answer: 6 units.