Powers and roots

6 questions | 15 marks

Equivalent correct methods earn the corresponding marks.

1. Solve $z^2 = -9$. [2 marks]

Use arguments $\frac{\pi + 2k\pi}{2}$ for $k = 0, 1$ and modulus 3.

Answer: $z = 3i$ or $z = -3i$.

2. List the cube roots of 8 in polar form. [2 marks]

The angles are $\frac{2k\pi}{3}$ for $k = 0, 1, 2$.

Answer: $2,\ 2\text{cis}(\frac{2\pi}{3}),\ 2\text{cis}(\frac{4\pi}{3})$.

3. Find all solutions of $z^4 = -16$. [3 marks]

The modulus is 2. Arguments come from $\frac{\pi + 2k\pi}{4}$.

Answer: $2\text{cis}(\frac{\pi}{4} + \frac{k\pi}{2}),\ k = 0, 1, 2, 3$.

4. Write the roots of $z^3 = 8i$ in principal-argument polar form. [3 marks]

Angles from $\frac{\pi}{6} + \frac{2k\pi}{3}$ are $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$; reduce the last by 2π.

Answer: $2\text{cis}(\frac{\pi}{6}),\ 2\text{cis}(\frac{5\pi}{6}),\ 2\text{cis}(-\frac{\pi}{2})$.

5. Explain why $k = 4$ gives no new root in $z_k = 2\text{cis}(\frac{\pi}{8} + \frac{k\pi}{2})$. [2 marks]

The angle for $k = 4$ is $\frac{\pi}{8} + 2\pi$, the same direction as $\frac{\pi}{8}$.

Answer: It repeats the $k = 0$ root.

6. The three cube roots of a nonzero number form an equilateral triangle. Explain using arguments. [3 marks]

Equal central angles on one circle give equal chord lengths, so all three sides are equal.

Answer: Adjacent roots differ in angle by $\frac{2\pi}{3}$ and have equal modulus.