De Moivre's theorem

6 questions | 14 marks

Equivalent correct methods earn the corresponding marks.

1. Calculate $(\sqrt{3}+i)^3$. [2 marks]

$\sqrt{3}+i=2\text{cis}(\frac{\pi}{6})$, so its cube is $8\text{cis}(\frac{\pi}{2})=8i$.

Answer: $8i$.

2. Calculate $(1-i)^6$. [2 marks]

$1-i=\sqrt{2}\text{cis}(-\frac{\pi}{4})$. The sixth power is $8\text{cis}(-\frac{3\pi}{2})=8i$.

Answer: $8i$.

3. Write $(2\text{cis}(\frac{\pi}{3}))^{-2}$ in polar form. [2 marks]

Raise the nonzero modulus to -2 and multiply the angle by -2.

Answer: $\frac{1}{4}\text{cis}(-\frac{2\pi}{3})$.

4. Use de Moivre and a square expansion to obtain a formula for $\cos 2\theta$. [3 marks]

The real part of $(\cos\theta+i\sin\theta)^2$ is $\cos^2\theta-\sin^2\theta$; de Moivre gives real part $\cos 2\theta$.

Answer: $\cos 2\theta=\cos^2\theta-\sin^2\theta$.

5. Show that $(\text{cis}(\frac{\pi}{5}))^{10}=1$. [2 marks]

De Moivre gives $\text{cis}(\frac{10\pi}{5})=\text{cis}(2\pi)=1$.

Answer: The tenth power is 1.

6. Find the least positive integer n for which $(\text{cis}(\frac{2\pi}{7}))^n=1$. [3 marks]

We require $\frac{2n\pi}{7}=2k\pi$, so $n=7k$. The least positive integer is 7.

Answer: $n=7$.