Multiplication as rotation/dilation

6 questions | 14 marks

Equivalent correct methods earn the corresponding marks.

1. Find the image of $3-2i$ under multiplication by i. [2 marks]

$i(3-2i)=3i-2i^2=2+3i$.

Answer: $2+3i$.

2. Describe the map $w=-3z$. [2 marks]

$-3=3\text{cis}\,\pi$.

Answer: Dilation of scale factor 3 and rotation through π about the origin.

3. A robot step is represented by $z=2+i$. A turn-and-scale instruction multiplies it by $1+i$. Find the new step. [3 marks]

$(1+i)(2+i)=2+3i+i^2=1+3i$. The instruction scales by $\sqrt{2}$ and rotates by $\frac{\pi}{4}$.

Answer: $1+3i$.

4. Find the multiplier taking 2 to $-2+2\sqrt{3}i$. [2 marks]

Divide image by original: $a=\frac{-2+2\sqrt{3}i}{2}$. Its modulus is 2 and quadrant-II angle is $\frac{2\pi}{3}$.

Answer: $-1+\sqrt{3}i=2\text{cis}(\frac{2\pi}{3})$.

5. Find the inverse of $w=2iz+3$. [3 marks]

Subtract 3 first, then divide by $2i$.

Answer: $z=\frac{w-3}{2i}=-\frac{i}{2}(w-3)$.

6. Two instructions multiply successively by $2i$ and $-\frac{i}{2}$. What is the combined transformation? [2 marks]

The combined multiplier is $(2i)(-\frac{i}{2})=-i^2=1$.

Answer: The identity transformation.