Mathematical induction - Practice test 6

Name: _____ Date: _____

10 questions | 56 marks | 75 minutes

Proof structure, distinct summation identities, integer divisibility and criticism of arguments.

No calculator needed.

1. A proof establishes $P(5)$ and $P(k)$ implies $P(k+1)$ for every integer k at least 5. [4 marks]

(a) State the exact range established by the proof. [2 marks]

(b) Explain whether $P(4)$ follows, and what is needed to claim all positive integers. [2 marks]

2. Prove $n^5 - n$ is divisible by 5 for integers $n \geq 1$. [6 marks]

(a) Verify the first case and state a precise induction hypothesis. [2 marks]

(b) Establish the successor and conclude the stated domain. [4 marks]

3. Prove $\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$ for integers $n \geq 1$. [6 marks]

(a) Verify the first case and state a precise induction hypothesis. [2 marks]

(b) Establish the successor and conclude the stated domain. [4 marks]

4. Prove $\sum_{r=1}^{n} (1 + 2 + \cdots + r) = \frac{n(n+1)(n+2)}{6}$ for integers $n \geq 1$. [6 marks]

(a) Verify the first case and state a precise induction hypothesis. [2 marks]

(b) Establish the successor and conclude the stated domain. [4 marks]

5. Prove $10^n - 3^n$ is divisible by 7 for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

6. The first case has been checked for $7^n - (-1)^n$ is divisible by 8. Complete the remaining induction argument. [6 marks]

(a) State exactly what is assumed at an arbitrary kth case. [1 mark]

(b) Derive the successor case. Identify where the hypothesis is used. [4 marks]

(c) Explain why the checked base plus this implication covers all positive integers. [1 mark]

7. Let $W_n = \sum_{r=1}^{n} r^2 2^r$ for positive integers n. [6 marks]

(a) Check the base case and state the hypothesis for $W_n = (n^2 - 2n + 3)2^{n+1} - 6$. [2 marks]

(b) Prove the successor formula and hence calculate W_4. [4 marks]

8. Consider $R_n = \sum_{r=1}^{n} \frac{r^2}{4r^2 - 1}$ for positive integers n. [6 marks]

(a) Verify the first case of $R_n = \frac{n(n+1)}{2(2n+1)}$, and state the induction hypothesis and new term. [2 marks]

(b) Establish the successor case and conclude. [4 marks]

9. A computer checked $\prod_{r=1}^{n} \frac{2r-1}{2r+1} = \frac{1}{2n+1}$ for the first one hundred positive integers. Replace that numerical evidence with a proof. [6 marks]

(a) Explain the limitation of the computer check and verify a valid starting case. [2 marks]

(b) State the hypothesis and derive the next case with exact algebra. [4 marks]

10. A divisibility proof concludes $A(k+1) = d(q+\frac{1}{2})$ with integer q. Explain the problem. [4 marks]

(a) Explain why $q+\frac{1}{2}$ is not an integer multiplier and why this fails to establish divisibility. [2 marks]

(b) State the required form of the conclusion. [2 marks]