Mathematical induction - Practice test 5

Name: _____ Date: _____

10 questions | 57 marks | 75 minutes

Proof structure, distinct summation identities, integer divisibility and criticism of arguments.

No calculator needed.

1. A triangular stack has n rows; row r contains exactly 2r tiles. [5 marks]

(a) Find the total tile count through row n. [2 marks]

(b) If the total formula holds through row k, write the next row count and obtain the next total. [3 marks]

2. Prove $\sum_{r=1}^{n} r(r+1)(r+2) = \frac{n(n+1)(n+2)(n+3)}{4}$ for integers $n \geq 1$. [6 marks]

(a) Verify the first case and state a precise induction hypothesis. [2 marks]

(b) Establish the successor and conclude the stated domain. [4 marks]

3. Prove $\prod_{r=1}^{n} \frac{r+2}{r} = \frac{(n+1)(n+2)}{2}$ for integers $n \geq 1$. [6 marks]

(a) Verify the first case and state a precise induction hypothesis. [2 marks]

(b) Establish the successor and conclude the stated domain. [4 marks]

4. Prove $4^n + 6n - 1$ is divisible by 9 for integers $n \geq 1$. [6 marks]

(a) Verify the first case and state a precise induction hypothesis. [2 marks]

(b) Establish the successor and conclude the stated domain. [4 marks]

5. Prove $11^n - 3^n$ is divisible by 8 for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

6. Consider the alternating sum $S_n = \sum_{r=1}^{n} (-1)^{r-1} r^2$. [6 marks]

(a) Calculate S_1, S_2 and S_3, then check the formula $S_n = \frac{(-1)^{n-1} n(n+1)}{2}$ at the first case. [2 marks]

(b) Prove the formula by induction, taking care with the sign of the new term. [4 marks]

7. The first case has been checked for $\sum_{r=1}^{n}(2r-1)^2=\frac{n(4n^2-1)}{3}$. Complete the remaining induction argument. [6 marks]

(a) State exactly what is assumed at an arbitrary kth case. [1 mark]

(b) Derive the successor case. Identify where the hypothesis is used. [4 marks]

(c) Explain why the checked base plus this implication covers all positive integers. [1 mark]

8. For positive integers n, let $D_n=\sum_{r=1}^{n}\frac{r}{2^r}$. [6 marks]

(a) Verify the proposed formula $D_n=2-\frac{n+2}{2^n}$ at $n=1$, and state an exact induction hypothesis. [2 marks]

(b) Prove the successor and use the formula to calculate D_6 exactly. [4 marks]

9. For positive integers n, define the product of the first n odd numbers by $O_n = 1 \cdot 3 \cdot 5 \cdots (2n-1)$. [6 marks]

(a) Explain the identity $(2n)! = 2^n n!\, O_n$ by grouping the even and odd factors. [2 marks]

(b) Independently prove $O_n = \frac{(2n)!}{2^n n!}$ by induction. [4 marks]

10. A sum of odd numbers proof adds $2k-1$ to the assumed total k^2. Correct the next term. [4 marks]

(a) Identify the next odd term that must be added and explain why $2k-1$ repeats an old term. [2 marks]

(b) Show the corrected successor total. [2 marks]