Mathematical induction - Practice test 3

Name: _____ Date: _____

10 questions | 57 marks | 75 minutes

Proof structure, distinct summation identities, integer divisibility and criticism of arguments.

No calculator needed.

1. The numbers $n(n+1)$ are even for positive integers n. [4 marks]

(a) Give a direct argument. [2 marks]

(b) Explain why a correct direct proof does not also need induction. [2 marks]

2. Prove $\sum_{r=1}^{2n}(-1)^{r-1}r=-n$ for integers $n\geq 1$. [6 marks]

(a) Verify the first case and state a precise induction hypothesis. [2 marks]

(b) Establish the successor and conclude the stated domain. [4 marks]

3. Prove $\sum_{r=1}^{n} r\,r! = (n+1)! - 1$ for integers $n \geq 1$. [6 marks]

(a) Verify the first case and state a precise induction hypothesis. [2 marks]

(b) Establish the successor and conclude the stated domain. [4 marks]

4. Prove $\prod_{r=2}^{n} \frac{r^2}{r^2-1} = \frac{2n}{n+1}$ for integers $n \geq 2$. [6 marks]

(a) Verify the first case and state a precise induction hypothesis. [2 marks]

(b) Establish the successor and conclude the stated domain. [4 marks]

5. Prove $7^n - 1$ is divisible by 6 for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

6. Prove $7^n - (-1)^n$ is divisible by 8 for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

7. A student uses $S_k = \frac{k}{k+1}$ for $S_n = \sum_{r=1}^{n} \frac{1}{r(r+1)}$, then writes $S_{k+1} = S_k + \frac{1}{k(k+1)}$. [6 marks]

(a) Identify the error and write the correct successor expression. [2 marks]

(b) Complete the corrected successor algebra, and give the exact value of the sum from $r = 2$ to $r = n$, for n at least 2. [4 marks]

8. Prove $\sum_{r=1}^{n} r(3r+1) = n(n+1)^2$ for every positive integer n. [6 marks]

(a) Check the base case and state the full induction hypothesis. [2 marks]

(b) Prove the successor case and conclude. [4 marks]

9. Let $P_n = \prod_{r=1}^{n} \frac{r(r+2)}{(r+1)^2}$ for positive integers n. [6 marks]

(a) Calculate P_1 and P_2, and conjecture a formula for P_n. [2 marks]

(b) Prove the conjecture by induction, identifying the new factor and any denominator restrictions. [4 marks]

10. The step is proved only for k at least 5 and the student checks $P(3)$. What is missing to establish every n at least 3? [5 marks]

(a) Identify the missing cases needed before the valid successor chain can cover every integer $n \geq 3$. [3 marks]

(b) If the missing cases are also proved, explain why the entire claimed domain follows. [2 marks]