Mathematical induction - Practice test 2

Name: _____ Date: _____

10 questions | 57 marks | 75 minutes

Proof structure, distinct summation identities, integer divisibility and criticism of arguments.

No calculator needed.

1. A student wants to prove a formula for every integer n at least 2. [5 marks]

(a) State the required starting case and explain the role of an arbitrary k. [2 marks]

(b) A proposed implication moves from k to $k+2$. Give enough starting cases and explain why. [3 marks]

2. Prove $\sum_{r=1}^{n}(3r-1)=\frac{3n(n+1)}{2}-n$ for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

3. Prove $\sum_{r=1}^{n} r^3 = [\frac{n(n+1)}{2}]^2$ for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

4. Prove $1 + 2 + 2^2 + \cdots + 2^{n-1} = (2^n - 1)$ for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

5. Prove $8^n - 3^n$ is divisible by 5 for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

6. Prove $n^3 + 2n$ is divisible by 3 for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

7. Prove $\sum_{r=1}^{n} r\,2^{r-1} = (n-1)2^n + 1$ for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

8. Prove $\prod_{r=1}^{n} \frac{2r-1}{2r+1} = \frac{1}{2n+1}$ for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

9. Prove $\sum_{r=1}^{n} r(r+1) = \frac{n(n+1)(n+2)}{3}$ for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

10. Why do correct checks for the first one hundred positive integers not constitute induction? [4 marks]

(a) Explain why checking finitely many cases cannot establish the successor implication for an arbitrary positive integer. [2 marks]

(b) Which induction obligation has already been met, and which remains? [2 marks]