Mathematical induction - Practice test 1

Name: _____ Date: _____

10 questions | 58 marks | 75 minutes

Proof structure, distinct summation identities, integer divisibility and criticism of arguments.

No calculator needed.

1. A paving design starts with four blocks. Each successive stage contains exactly three more blocks than the preceding stage. [5 marks]

(a) Find a rule for the number in stage n. [2 marks]

(b) Suggest the cumulative total through stage n, and explain how it could be proved. [3 marks]

2. Prove $1+2+\cdots+n=\frac{n(n+1)}{2}$ for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

3. Prove $1+3+\cdots+(2n-1)=n^2$ for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

4. Prove $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$ for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

5. Prove $1 + 3 + 3^2 + \cdots + 3^{n-1} = \frac{3^n - 1}{2}$ for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

6. Prove $5^n - 2^n$ is divisible by 3 for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

7. Prove $n^3 - n$ is divisible by 6 for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

8. Prove $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$ for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

9. Prove $\prod_{r=1}^{n} \frac{r+1}{r} = n+1$ for all integers $n \geq 1$. [6 marks]

(a) Verify the first case and state the induction hypothesis. [2 marks]

(b) Prove the next case and conclude. [4 marks]

10. A student assumes the result at $k+1$ and rearranges it to the hypothesis at k. Explain the logical gap. [5 marks]

(a) Identify which implication the written argument proves, and which implication induction requires. [3 marks]

(b) Describe a valid repair of the proof direction. [2 marks]