Recognising incomplete or invalid induction arguments

6 questions | 15 marks

Equivalent correct methods earn the corresponding marks.

1. A student assumes the result at $k+1$ and rearranges it to the hypothesis at k. Explain the logical gap. [3 marks]

Derive $P(k+1)$ from $P(k)$.

If reversing algebra, explicitly justify that each step is reversible.

Answer: The direction written is the reverse of the required implication.

2. Why do correct checks for the first one hundred positive integers not constitute induction? [2 marks]

An untested case could still fail.

A starting case and a general successor implication are both required.

Answer: There is no implication proved for an arbitrary integer.

3. The step is proved only for k at least 5 and the student checks $P(3)$. What is missing to establish every n at least 3? [3 marks]

The implication cannot be used at $k = 3$ or $k = 4$.

$P(5)$ is needed to start the valid chain.

Answer: $P(4)$ and $P(5)$.

4. Refute the claim that $n^2 + n + 41$ is prime for every positive integer n. [3 marks]

This allowed case is composite.

One counterexample refutes a universal claim.

Answer: $n = 41$ gives $41^2 + 41 + 41 = 41(43)$.

5. A sum of odd numbers proof adds $2k - 1$ to the assumed total k^2. Correct the next term. [2 marks]

The assumed sum already contains $2k - 1$.

The next odd number is $2(k + 1) - 1$.

Answer: It must add $2k + 1$.

6. A divisibility proof concludes $A(k + 1) = d(q + \frac{1}{2})$ with integer q. Explain the problem. [2 marks]

Integer divisibility requires an integer multiplier.

An arbitrary rational multiplier does not suffice.

Answer: The proposed multiplier is not an integer.