Practice worksheet | Mathematical induction

# Identities involving powers and integers

Name: _____ Date: _____

6 questions | 36 marks

**1.** Prove $1+2+2^2+\cdots+2^{n-1}=(2^n-1)$ for all integers $n \geq 1$. [6 marks]

**(a)** Verify the first case and state the induction hypothesis. [2 marks]

**(b)** Prove the next case and conclude. [4 marks]

**2.** Prove $1+3+3^2+\cdots+3^{n-1}=\frac{3^n-1}{2}$ for all integers $n \geq 1$. [6 marks]

**(a)** Verify the first case and state the induction hypothesis. [2 marks]

**(b)** Prove the next case and conclude. [4 marks]

**3.** Prove $\sum_{r=1}^{n} r\,2^{r-1} = (n-1)2^n + 1$ for all integers $n \geq 1$. [6 marks]

**(a)** Verify the first case and state the induction hypothesis. [2 marks]

**(b)** Prove the next case and conclude. [4 marks]

**4.** Prove $\prod_{r=1}^{n} \frac{r+1}{r} = n + 1$ for all integers $n \geq 1$. [6 marks]

**(a)** Verify the first case and state the induction hypothesis. [2 marks]

**(b)** Prove the next case and conclude. [4 marks]

**5.** Prove $\prod_{r=1}^{n} \frac{2r-1}{2r+1} = \frac{1}{2n+1}$ for all integers $n \geq 1$. [6 marks]

**(a)** Verify the first case and state the induction hypothesis. [2 marks]

**(b)** Prove the next case and conclude. [4 marks]

**6.** Prove $\sum_{r=1}^{n} (2r-1)^2 = \frac{n(4n^2-1)}{3}$ for all integers $n \geq 1$. [6 marks]

**(a)** Verify the first case and state the induction hypothesis. [2 marks]

**(b)** Prove the next case and conclude. [4 marks]