Constructing the induction step

6 questions | 14 marks

Equivalent correct methods earn the corresponding marks.

1. Simplify $k^2 + 2k + 1$ and explain its induction use. [2 marks]

The new odd term is $2k + 1$.

Add it to the assumed odd-number sum k^2.

Answer: $(k+1)^2$.

2. Given $S_k = \frac{k(k+1)}{2}$, derive S_{k+1} when the next term is $k+1$. [3 marks]

$S_{k+1} = S_k + (k+1)$.

Substitute and factor $(k+1)(\frac{k}{2}+1)$.

Answer: $S_{k+1} = \frac{(k+1)(k+2)}{2}$.

3. What term extends $\sum_{r=1}^{k} r^2$ to the sum ending at $k+1$? [1 mark]

Substitute the new index into the entire term.

Answer: $(k+1)^2$.

4. If $\prod_{r=1}^{k} \frac{r+1}{r} = k+1$, find the next product. [3 marks]

Multiply by $\frac{k+2}{k+1}$.

Cancel the nonzero common factor.

Answer: $k+2$.

5. A student writes the target at $k+1$ immediately after the hypothesis at k. What is missing? [2 marks]

For a sum, expose the next term.

Use the hypothesis only for the old sum.

Answer: A justified bridge from the new expression to the old expression.

6. For $A_n = 5^n - 1$, establish $A_{k+1} = 5A_k + 4$. [3 marks]

Expanding gives $5^{k+1} - 5 + 4$.

The identity connects the new case to the old case.

Answer: $5^{k+1} - 1 = 5(5^k - 1) + 4$.