Base case and induction hypothesis

6 questions | 13 marks

Equivalent correct methods earn the corresponding marks.

1. Check the base case for $1+3+\cdots+(2n-1)=n^2$, for $n \geq 1$. [2 marks]

Both sides equal 1.

Answer: At $n=1$, $1=1^2$.

2. State the induction hypothesis for $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$. [2 marks]

Use k consistently in every position.

Do not assume the next case.

Answer: For arbitrary integer $k \geq 1$, assume $\sum_{r=1}^{k} r^2 = \frac{k(k+1)(2k+1)}{6}$.

3. Check the first case of 3^n-1 even for integers $n \geq 0$. [2 marks]

The smallest allowed integer is 0.

Zero is even.

Answer: At $n=0$, $3^0-1=0=2(0)$.

4. For positive integers n, write the induction hypothesis for n^3-n divisible by six. [2 marks]

The integer condition on q expresses divisibility.

Answer: Assume $k^3-k=6q$ for some integer q and arbitrary integer $k \geq 1$.

5. A theorem is stated for n at least 4, but the proof starts at $n=5$. Which case is omitted? [2 marks]

The successor implication cannot run backward.

Answer: $n=4$.

6. Why is an induction hypothesis not an assumption that the whole theorem is true? [3 marks]

$P(k)$ is assumed temporarily for arbitrary k.

The base case and successor implication together establish every required case.

Answer: It is the premise in one conditional implication.