Complex numbers - reasoning test

Name: _____ Date: _____

10 questions | 56 marks | 75 minutes

Use component equations, geometric constraints and short proofs to justify complex-number results.

No calculator needed.

1. Let $z = -2 - 3i$. [5 marks]

(a) Find $-\overline{z}$ and $\overline{-z}$. [3 marks]

(b) Explain why the equality holds for every complex z. [2 marks]

2. Consider $(2+i)z = 5$. [5 marks]

(a) Solve for z. [2 marks]

(b) Use moduli to check the magnitude of your result. [3 marks]

3. Use the cycle of powers of i. [5 marks]

(a) Evaluate $i^{25} - i^{14} + i^{8}$. [3 marks]

(b) Find i^{-1} and explain why it is not i. [2 marks]

4. A complex number satisfies $z + 2\overline{z} = 9 - 4i$. [6 marks]

(a) Find z by setting $z = a + bi$. [3 marks]

(b) Find $\frac{z}{\overline{z}}$ in Cartesian form. [3 marks]

5. A monic real quadratic $z^2 + pz + q = 0$ has a root $2i$. [5 marks]

(a) Find p and q. [3 marks]

(b) For a nonzero real b, state the monic real quadratic with roots $\pm bi$. [2 marks]

6. Let $p(z) = z^2 + az + b$ with real a, b, and suppose $p(1 + 2i) = 0$. [6 marks]

(a) Use substitution and component equations to find a, b. [4 marks]

(b) Verify that the conjugate is also a root. [2 marks]

7. A quadratic $q(z)$ has roots $-1 \pm i$ and satisfies $q(0) = 6$. [6 marks]

(a) Find its leading coefficient and expanded expression. [4 marks]

(b) Explain why the roots alone did not determine the leading coefficient. [2 marks]

8. Point Z represents $z = a + 2i$, with real a. It is equally far from 1 and $-3 + 4i$. [6 marks]

(a) Form an equation in a using squared distances. [3 marks]

(b) Find z and the common distance. [3 marks]

9. Take $z = a + bi$ and $w = c + di$, with real components. [6 marks]

(a) Express the real and imaginary components of zw. [2 marks]

(b) Prove $|zw| = |z||w|$. [4 marks]

10. A complex number z has real part 2 and is at distance $\sqrt{10}$ from the point 1. [6 marks]

(a) Find all possible values of z. [3 marks]

(b) Find the monic real quadratic whose roots are these two values. [3 marks]