Complex numbers - applications test

Name: _____ Date: _____

10 questions | 57 marks | 70 minutes

Use fully specified position, calibration and electrical-style models; connect each model to exact Cartesian arithmetic and geometry.

No calculator needed.

1. A drone's horizontal map uses complex coordinates in metres. It starts at $-2+i$ and moves in a straight line by $5-3i$. [5 marks]

(a) Find the final position. [2 marks]

(b) Find the distance travelled and the final distance from the origin. [3 marks]

2. A supplied circuit model defines the complex current by $I = \frac{V}{Z}$. Take $V = 10$ and $Z = 3 + 4i$. No physics knowledge is required. [6 marks]

(a) Find I in Cartesian form. [3 marks]

(b) The model calls $|I|$ the current magnitude. Find it and check using $\frac{|V|}{|Z|}$. [3 marks]

3. A design calculation requires the roots of $\lambda^2 - 4\lambda + 13 = 0$. [6 marks]

(a) Find the two roots exactly. [3 marks]

(b) Find their sum and product, and explain why both results are real. [3 marks]

4. A signal is represented by $z = t + 2i$, where t is a positive real number. Its magnitude is 5. [5 marks]

(a) Find t exactly. [3 marks]

(b) Find $z + \overline{z}$ and $z\overline{z}$. [2 marks]

5. A calibration rule sends an input z to output $w = (1 - i)z$. [6 marks]

(a) Find the output for $z = 3 + 2i$. [3 marks]

(b) An output is $w = 4 + 2i$. Recover its input. [3 marks]

6. A measurement model has unknown real constants a, b satisfying $(a + bi)(1 + i) = -1 + 7i$. [6 marks]

(a) Form and solve two real equations for a, b. [4 marks]

(b) Find $|a + bi|$. [2 marks]

7. A real quadratic $p(z)$ has leading coefficient 2 and a root $-1+3i$. [6 marks]

(a) Find $p(z)$ in expanded form. [4 marks]

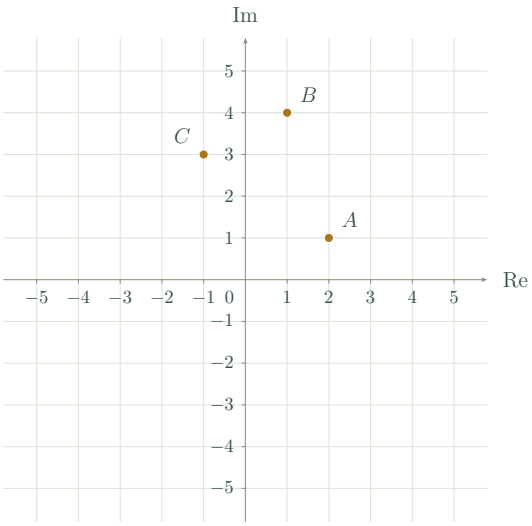
(b) Find $p(0)$ and $p(1)$. [2 marks]

8. A student writes $\frac{1}{2-3i} = \frac{2-3i}{13}$. [6 marks]

(a) Multiply the proposed answer by $2-3i$ to show the error. [3 marks]

(b) Give the correct reciprocal and explain the missing step. [3 marks]

9. A parallelogram has successive complex vertices 0, $2+i$, $1+4i$ and $-1+3i$. [6 marks]



(a) Show that the two routes from 0 to the opposite vertex agree. [2 marks]

(b) Find its centre and the lengths of both diagonals. [4 marks]

10. A geometric rule maps every complex point z to iz. [5 marks]

(a) Find the image of $2-3i$ and compare their distances from the origin. [3 marks]

(b) Find the result after applying the rule four times and explain algebraically. [2 marks]