Complex numbers - applications test

10 questions | 57 marks | 70 minutes

Use fully specified position, calibration and electrical-style models; connect each model to exact Cartesian arithmetic and geometry.

Equivalent correct methods earn the corresponding marks.

1. A drone's horizontal map uses complex coordinates in metres. It starts at $-2+i$ and moves in a straight line by $5-3i$. [5 marks]

(a) Find the final position. [2 marks]

Add position and displacement: $(-2+i)+(5-3i)=3-2i$.

Answer: $3-2i$ metres.

(b) Find the distance travelled and the final distance from the origin. [3 marks]

The move has components $(5,\,-3)$, giving length $\sqrt{25+9}$.

The final position has components $(3,\,-2)$, giving length $\sqrt{9+4}$.

Answer: Travelled $\sqrt{34}$ metres; final origin distance $\sqrt{13}$ metres.

2. A supplied circuit model defines the complex current by $I=\frac{V}{Z}$. Take $V=10$ and $Z=3+4i$. No physics knowledge is required. [6 marks]

(a) Find I in Cartesian form. [3 marks]

$I=\frac{10(3-4i)}{3^2+4^2}$.

Divide $30-40i$ by 25.

Answer: $I=\frac{6}{5}-\frac{8}{5}i$.

(b) The model calls $|I|$ the current magnitude. Find it and check using $\frac{|V|}{|Z|}$. [3 marks]

$\sqrt{\left(\frac{6}{5}\right)^2+\left(-\frac{8}{5}\right)^2}=\sqrt{\frac{100}{25}}=2$.

Also $\frac{|V|}{|Z|}=\frac{10}{5}=2$.

Answer: $|I|=2$.

3. A design calculation requires the roots of $\lambda^2 - 4\lambda + 13 = 0$. [6 marks]

(a) Find the two roots exactly. [3 marks]

Complete the square: $(\lambda - 2)^2 = -9$.

Answer: $\lambda = 2 \pm 3i$.

(b) Find their sum and product, and explain why both results are real. [3 marks]

The roots are conjugates, so imaginary components cancel in the sum.

The product is $2^2 + 3^2 = 13$.

Answer: Sum 4; product 13.

4. A signal is represented by $z = t + 2i$, where t is a positive real number. Its magnitude is 5. [5 marks]

(a) Find t exactly. [3 marks]

$|z|^2 = t^2 + 4 = 25$.

$t^2 = 21$ and positivity selects $\sqrt{21}$.

Answer: $t = \sqrt{21}$.

(b) Find $z + \bar{z}$ and $z\bar{z}$. [2 marks]

The sum is twice the real component.

The product is the squared modulus.

Answer: $2\sqrt{21}$ and 25.

5. A calibration rule sends an input z to output $w = (1 - i)z$. [6 marks]

(a) Find the output for $z = 3 + 2i$. [3 marks]

Expand: $3 + 2i - 3i - 2i^2 = 5 - i$.

Answer: $w = 5 - i$.

(b) An output is $w = 4 + 2i$. Recover its input. [3 marks]

$z = \frac{4 + 2i}{1 - i}$.

Multiply by $\frac{1 + i}{1 + i}$: numerator $2 + 6i$, denominator 2.

Answer: $z = 1 + 3i$.

6. A measurement model has unknown real constants a, b satisfying $(a+bi)(1+i) = -1+7i$. [6 marks]

(a) Form and solve two real equations for a, b. [4 marks]

The product is $(a-b)+(a+b)i$.

So $a-b=-1$ and $a+b=7$.

Adding gives $2a=6$; then $b=4$.

Answer: $a=3$, $b=4$.

(b) Find $|a+bi|$. [2 marks]

$|3+4i| = \sqrt{9+16} = 5$.

Answer: 5.

7. A real quadratic $p(z)$ has leading coefficient 2 and a root $-1+3i$. [6 marks]

(a) Find $p(z)$ in expanded form. [4 marks]

The other root is $-1-3i$.

$p(z) = 2[(z+1)^2+9]$.

Expand.

Answer: $p(z) = 2z^2+4z+20$.

(b) Find $p(0)$ and $p(1)$. [2 marks]

Substitute the two real inputs into the expanded expression.

Answer: 20 and 26.

8. A student writes $\frac{1}{2-3i} = \frac{2-3i}{13}$. [6 marks]

(a) Multiply the proposed answer by $2-3i$ to show the error. [3 marks]

$(2-3i)^2 = 4-12i+9i^2 = -5-12i$.

Answer: The product is $\frac{-5-12i}{13}$, not 1.

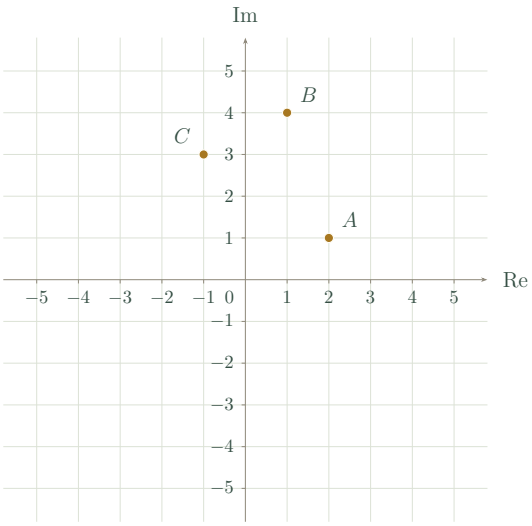
(b) Give the correct reciprocal and explain the missing step. [3 marks]

Use the conjugate $2+3i$, not the original denominator.

$(2-3i)(2+3i) = 13$.

Answer: $\frac{2+3i}{13}$.

9. A parallelogram has successive complex vertices 0, $2+i$, $1+4i$ and $-1+3i$. [6 marks]



(a) Show that the two routes from 0 to the opposite vertex agree. [2 marks]

The two side displacements add to the opposite position.

Complex addition is commutative, so either route reaches the same point.

Answer: $(2+i)+(-1+3i)=1+4i$.

(b) Find its centre and the lengths of both diagonals. [4 marks]

The centre is $\frac{1+4i}{2}$.

One diagonal has displacement $1+4i$.

The other has displacement $(2+i)-(-1+3i)=3-2i$.

Answer: Centre $\frac{1}{2}+2i$; diagonals $\sqrt{17}$ and $\sqrt{13}$.

10. A geometric rule maps every complex point z to iz. [5 marks]

(a) Find the image of $2-3i$ and compare their distances from the origin. [3 marks]

$i(2-3i)=2i-3i^2=3+2i$.

Both squared distances equal 13.

Answer: The image is $3+2i$; both distances are $\sqrt{13}$.

(b) Find the result after applying the rule four times and explain algebraically. [2 marks]

Four applications multiply by $i^4=1$.

Thus the final point is $i^4z=z$.

Answer: The original point returns.