Vector interpretation of addition and conjugation

6 questions | 18 marks

Equivalent correct methods earn the corresponding marks.

1. Find $u+v$ and $u-v$ for $u=3-2i$ and $v=-1+i$. [3 marks]

Add or subtract the two real components and the two imaginary components.

Answer: $u+v=2-i$; $u-v=4-3i$.

2. Find the vector from $A=-2+i$ to $B=1-3i$. [2 marks]

$B-A=(1-3i)-(-2+i)=3-4i$.

Answer: $3-4i$.

3. Find the midpoint between $2+5i$ and $-4-i$. [3 marks]

Average components: $\frac{2-4}{2}=-1$ and $\frac{5-1}{2}=2$.

Answer: $-1+2i$.

4. Describe the effect of multiplying all complex numbers by -2. [3 marks]

Both coordinates double in magnitude and reverse sign.

Answer: A dilation by factor 2 about the origin followed by a half-turn.

5. Prove $z+\overline{z}$ is real and $z-\overline{z}$ has real part zero for every complex z. [4 marks]

Write the conjugate as $a-bi$.

Add and subtract componentwise; each result has the required zero component.

Answer: If $z=a+bi$, the expressions are $2a$ and $2bi$. The second is pure imaginary when $b\neq 0$, and is zero when $b=0$.

6. A parallelogram has successive vertices 0, z, $z+w$, w. For $z=4+i$ and $w=-1+2i$, find its centre. [3 marks]

The opposite vertices 0 and $z+w=3+3i$ have midpoint $\frac{3+3i}{2}$.

Diagonals of a parallelogram share their midpoint.

Answer: $\frac{3}{2}+\frac{3}{2}i$.