Conjugates and division

6 questions | 17 marks

Equivalent correct methods earn the corresponding marks.

1. Find the conjugate of $-5-2i$. [2 marks]

Keep the real part and reverse the imaginary component.

Answer: $-5+2i$.

2. Find $z\bar{z}$ for $z=4-3i$. [2 marks]

$z\bar{z}=4^2+(-3)^2=25$.

Answer: 25.

3. Simplify $\frac{2-i}{3+i}$. [3 marks]

Multiply by $\frac{3-i}{3-i}$.

The numerator is $6-2i-3i+i^2=5-5i$ and denominator is 10.

Answer: $\frac{1}{2}-\frac{1}{2}i$.

4. Simplify $\frac{5}{1+2i}$. [3 marks]

Multiply by $\frac{1-2i}{1-2i}$.

The denominator is 5, giving $\frac{5(1-2i)}{5}$.

Answer: $1-2i$.

5. Solve $(2-i)z=7+i$. [3 marks]

$z=\frac{7+i}{2-i}$.

Multiply by $\frac{2+i}{2+i}$: the numerator is $13+9i$ and denominator 5.

Answer: $z=\frac{13}{5}+\frac{9}{5}i$.

6. Show that $\frac{z}{\bar{z}}$ has real value only when $ab=0$, where $z=a+bi\neq 0$ and a,b are real. [4 marks]

$\frac{z}{\bar{z}}=\frac{(a+bi)^2}{a^2+b^2}$.

The numerator is a^2-b^2+2abi.

The denominator is positive, so its imaginary part is zero exactly when $2ab=0$.

Answer: The imaginary part is $\frac{2ab}{a^2+b^2}$, so it vanishes exactly when $ab=0$.