Sequences and series - Practice test 3

Name: _____ Date: _____

10 questions | 53 marks | 70 minutes

Interpreting data, convergence and practical models.

A scientific calculator may be used.

1. A pattern has the counts shown in the table. [5 marks]

Stage	1	2	3	4
Count	5	11	17	23

(a) Give a recurrence and an explicit rule. [3 marks]

(b) Find the total count across the first 10 stages. [2 marks]

2. The arithmetic sequence begins $40, 37, 34, \ldots$. [5 marks]

(a) Find its first negative term. [2 marks]

(b) Find the sum of all its positive terms. [3 marks]

3. An arithmetic sequence has $u_3 = 10, u_7 = 26$. Form a new list from its odd-positioned terms. [5 marks]

(a) Find the original rule. [2 marks]

(b) Find the first five terms and sum of the new list. [3 marks]

4. A real geometric sequence has $u_3 = 8, u_5 = 32$. [5 marks]

(a) Find all possible ratios and first terms. [3 marks]

(b) Explain why the supplied odd-index terms cannot decide the sign of r. [2 marks]

5. A geometric series starts at 6 and sums to 18 at infinity. [6 marks]

(a) Find its ratio and first three-term sum. [3 marks]

(b) Find the least number of terms for an omitted tail below 1. [3 marks]

6. A repeated process is modelled by $u_{n+1} = 0.75u_n + 5$, $u_0 = 4$. [5 marks]

(a) Find the equilibrium and an explicit rule. [3 marks]

(b) Find u_2 and explain whether values approach the equilibrium from above or below. [2 marks]

7. The decimal $0.\overline{27} = 0.272727\ldots$ can be written as a geometric series. [5 marks]

(a) Write the first three terms and ratio. [2 marks]

(b) Find the exact fraction. [3 marks]

8. A vehicle is worth AUD 18,000 initially. Model A loses AUD 2000 each year; model B loses 20% yearly. [6 marks]

(a) Find each predicted value after three years. [3 marks]

(b) Explain why neither model should be applied indefinitely without checking its assumptions. [3 marks]

9. Prove $2+4+\cdots+2n=n(n+1)$ for integers $n \geq 1$. [6 marks]

(a) Give the base case and hypothesis. [2 marks]

(b) Complete the implication and conclusion. [4 marks]

10. A student writes $S_\infty = \frac{a}{1-r}$ for $3+6+12+\cdots$ and obtains -3. [5 marks]

(a) Identify the invalid step. [2 marks]

(b) Use partial sums to explain why a negative total is impossible here. [3 marks]