Marking guide | Sequences and series

Sequences and series - Practice test 3

10 questions | 53 marks | 70 minutes

Interpreting data, convergence and practical models.

Equivalent correct methods earn the corresponding marks.

1. A pattern has the counts shown in the table. [5 marks]

Stage	1	2	3	4
Count	5	11	17	23

(a) Give a recurrence and an explicit rule. [3 marks]

Consecutive values differ by 6.

Answer: $u_1 = 5, u_{n+1} = u_n + 6;\ u_n = 6n - 1.$

(b) Find the total count across the first 10 stages. [2 marks]

$S_{10} = \frac{10(5 + 59)}{2} = 320.$

Answer: $320.$

2. The arithmetic sequence begins $40, 37, 34, \ldots$. [5 marks]

(a) Find its first negative term. [2 marks]

$u_n = 43 - 3n;\ u_{14} = 1, u_{15} = -2.$

Answer: $u_{15} = -2.$

(b) Find the sum of all its positive terms. [3 marks]

There are 14 positive terms; $\frac{14(40 + 1)}{2} = 287.$

Answer: $287.$

3. An arithmetic sequence has $u_3 = 10, u_7 = 26$. Form a new list from its odd-positioned terms. [5 marks]

(a) Find the original rule. [2 marks]

$4d = 16$, so $d = 4, a = 2$.

Answer: $u_n = 4n - 2$.

(b) Find the first five terms and sum of the new list. [3 marks]

Use original indices $1, 3, 5, 7, 9$; the new difference is 8.

Answer: $2, 10, 18, 26, 34$; sum 90.

4. A real geometric sequence has $u_3 = 8, u_5 = 32$. [5 marks]

(a) Find all possible ratios and first terms. [3 marks]

$r^2 = 4$ and $ar^2 = 8$.

Answer: $r = 2$ or -2; $a = 2$ in both cases.

(b) Explain why the supplied odd-index terms cannot decide the sign of r. [2 marks]

Positive and negative ratios of the same magnitude produce the same odd-index terms.

Answer: The relevant powers of r are even.

5. A geometric series starts at 6 and sums to 18 at infinity. [6 marks]

(a) Find its ratio and first three-term sum. [3 marks]

$\frac{6}{1-r} = 18$; $6 + 4 + \frac{8}{3} = \frac{38}{3}$.

Answer: $r = \frac{2}{3}$; $S_3 = \frac{38}{3}$.

(b) Find the least number of terms for an omitted tail below 1. [3 marks]

Tail $18\left(\frac{2}{3}\right)^n$; at $n = 7$ it is $\frac{256}{243} > 1$, at $n = 8$ it is $\frac{512}{729} < 1$.

Answer: 8.

6. A repeated process is modelled by $u_{n+1} = 0.75u_n + 5$, $u_0 = 4$. [5 marks]

(a) Find the equilibrium and an explicit rule. [3 marks]

$L = 0.75L + 5$; the initial deviation is -16.

Answer: $L = 20$; $u_n = 20 - 16(0.75)^n$.

(b) Find u_2 and explain whether values approach the equilibrium from above or below. [2 marks]

$20 - 16(0.75)^2 = 11$; the deviation remains negative and shrinks.

Answer: $u_2 = 11$; from below.

7. The decimal $0.\overline{27} = 0.272727\ldots$ can be written as a geometric series. [5 marks]

(a) Write the first three terms and ratio. [2 marks]

Each two-digit block is shifted two decimal places.

Answer: $\frac{27}{100} + \frac{27}{10000} + \frac{27}{1000000} + \cdots$; $r = \frac{1}{100}$.

(b) Find the exact fraction. [3 marks]

$\frac{\frac{27}{100}}{1 - \frac{1}{100}} = \frac{27}{99} = \frac{3}{11}$.

Answer: $\frac{3}{11}$.

8. A vehicle is worth AUD 18,000 initially. Model A loses AUD 2000 each year; model B loses 20% yearly. [6 marks]

(a) Find each predicted value after three years. [3 marks]

$18000 - 3(2000) = 12000$; $18000(0.8)^3 = 9216$.

Answer: 12000 and 9216.

(b) Explain why neither model should be applied indefinitely without checking its assumptions. [3 marks]

Each is a simplified model, not a guaranteed market value.

Answer: The linear model eventually predicts negative value; a fixed percentage may fail to reflect actual resale/maintenance conditions.

9. Prove $2 + 4 + \cdots + 2n = n(n+1)$ for integers $n \geq 1$. [6 marks]

(a) Give the base case and hypothesis. [2 marks]

The hypothesis is for arbitrary integer $k \geq 1$.

Answer: $2 = 1(2)$; assume $S_k = k(k+1)$.

(b) Complete the implication and conclusion. [4 marks]

Add the $(k+1)$th term $2(k+1)$ and factor.

Answer: $S_{k+1} = k(k+1) + 2(k+1) = (k+1)(k+2)$, hence true by induction.

10. A student writes $S_\infty = \frac{a}{1-r}$ for $3 + 6 + 12 + \cdots$ and obtains -3. [5 marks]

(a) Identify the invalid step. [2 marks]

The infinite-sum formula cannot be used.

Answer: The convergence condition fails because $r = 2$.

(b) Use partial sums to explain why a negative total is impossible here. [3 marks]

$S_n = 3(2^n - 1)$, which grows as n increases.

Answer: Every partial sum is positive and increases without bound.