Sequences and series - Practice test 2

10 questions | 56 marks | 70 minutes

Comparing rules, timing and proof structure.

Equivalent correct methods earn the corresponding marks.

1. Two lists are $A: 2, 5, 8, 11, \ldots$ and $B: 2, 6, 18, 54, \ldots$. [5 marks]

(a) Classify each and give its explicit rule. [3 marks]

Use a constant difference for A and quotient for B.

Answer: A arithmetic: $a_n = 3n - 1$; B geometric: $b_n = 2 \cdot 3^{n-1}$.

(b) Find the fourth-term sum for each. [2 marks]

$2 + 5 + 8 + 11 = 26$; $2 + 6 + 18 + 54 = 80$.

Answer: 26 and 80.

2. An arithmetic sequence has $u_3 = 11, u_8 = 31$. [5 marks]

(a) Find its rule. [2 marks]

$5d = 20$ gives $d = 4$ and $a = 3$.

Answer: $u_n = 4n - 1$.

(b) Find the first term greater than 100 and its index. [3 marks]

$4n - 1 > 100$ gives $n > 25.25$.

Answer: $u_{26} = 103$.

3. Four arithmetic means are inserted between 7 and 32. [4 marks]

(a) Find the means. [2 marks]

There are five equal intervals, so $d = \frac{32-7}{5} = 5$.

Answer: $12, 17, 22, 27$.

(b) Find the sum of all six terms. [2 marks]

$\frac{6(7+32)}{2} = 117$.

Answer: 117.

4. A positive geometric sequence has $u_2 = 12, u_5 = 96$. [5 marks]

(a) Find a and r. [2 marks]

$r^3 = \frac{96}{12} = 8$; $ar = 12$.

Answer: $a = 6, r = 2$.

(b) Find the sum of its first five terms. [3 marks]

$\frac{6(2^5-1)}{2-1} = 186$.

Answer: 186.

5. Consider $4 - 2 + 1 - \frac{1}{2} + \cdots$. [6 marks]

(a) Find its first six-term sum. [3 marks]

$S_6 = \frac{4\left[1-\left(-\frac{1}{2}\right)^6\right]}{\frac{3}{2}} = \frac{21}{8}$.

Answer: $\frac{21}{8} = 2.625$.

(b) Find the infinite sum and compare it with the six-term sum. [3 marks]

$\frac{4}{\frac{3}{2}} = \frac{8}{3}$; $\frac{8}{3} - \frac{21}{8} = \frac{1}{24}$.

Answer: $\frac{8}{3}$; it exceeds S_6 by $\frac{1}{24}$.

6. The geometric series $1+\frac{1}{3}+\frac{1}{9}+\cdots$ is approximated by its first n terms. [6 marks]

(a) Find its infinite sum and tail after n terms. [3 marks]

The next term is 3^{-n}; divide it by $1-\frac{1}{3}$.

Answer: $\frac{3}{2}$; tail $\left(\frac{3}{2}\right)3^{-n}$.

(b) Find the least n for a tail below 0.001. [3 marks]

At 6 terms the tail is $\frac{1}{486}>0.001$; at 7 it is $\frac{1}{1458}<0.001$.

Answer: $n=7$.

7. A debt is AUD 1500 initially. Each month 2% interest is added, then AUD 200 is paid. [6 marks]

(a) Calculate the first two updated balances. [2 marks]

$1.02(1500)-200=1330;\ 1.02(1330)-200=1156.6$.

Answer: 1330 and 1156.60.

(b) Find a closed formula valid before the debt is repaid. [4 marks]

Equilibrium L satisfies $L=1.02L-200$, so $L=10000$; the initial deviation is -8500.

Answer: $B_n=10000-8500(1.02)^n$.

8. A painter covers 6 square metres on day 1 and 2 extra square metres each later day. [6 marks]

(a) Write the total area after n days. [3 marks]

$\frac{n[12+2(n-1)]}{2}=n(n+5)$.

Answer: $S_n=n(n+5)$ square metres.

(b) Find the first day by which the total exceeds 100 square metres. [3 marks]

$S_7=84;\ S_8=104$.

Answer: Day 8.

9. For fixed real a, d, let $S_n = a + (a+d) + \cdots + [a + (n-1)d]$. [7 marks]

(a) State the arithmetic-sum identity and check $n = 1$. [2 marks]

Substitute 1 in the formula.

Answer: $S_n = \frac{n[2a + (n-1)d]}{2}$; the base is $a = a$.

(b) Prove the identity by induction. [5 marks]

Assume $S_k = \frac{k[2a + (k-1)d]}{2}$.

Add $a + kd$ and factor to obtain $\frac{(k+1)(2a + kd)}{2}$; conclude.

Answer: True for every integer $n \geq 1$.

10. An account begins at AUD 500. Compare model $A_{n+1} = 1.1A_n + 50$ with model $C_{n+1} = 1.1(C_n + 50)$. [6 marks]

(a) Explain the timing difference. [2 marks]

The new deposit earns interest in the second model.

Answer: A adds the deposit after interest; C adds it before interest.

(b) Find both balances after two periods and their difference. [4 marks]

$A_1 = 600, A_2 = 710$; $C_1 = 605, C_2 = 720.5$.

Answer: $A_2 = 710$, $C_2 = 720.50$; difference 10.50.