# Induction proofs of simple sums and arithmetic/geometric-series results

6 questions | 19 marks

Equivalent correct methods earn the corresponding marks.

**1.** Prove $3+7+\cdots+(4n-1)=n(2n+1)$. [4 marks]

Base: $3=1(3)$. Assume $S_k=k(2k+1)$.

Add $4k+3$ to get $2k^2+5k+3=(k+1)(2k+3)$; conclude.

**Answer:** True for integers $n\geq 1$.

**2.** Prove $1+3+\cdots+3^{n-1}=\frac{3^n-1}{2}$. [4 marks]

Base: $1=\frac{3-1}{2}$. Assume $S_k=\frac{3^k-1}{2}$.

Add $3^k$ to get $\frac{3^{k+1}-1}{2}$; conclude.

**Answer:** True for integers $n\geq 1$.

**3.** State the next term and required restriction in a proof of $a+ar+\cdots+ar^{n-1}=\frac{a(1-r^n)}{1-r}$. [2 marks]

The denominator must not be zero.

**Answer:** Next term $ar^k$; restriction $r\neq 1$.

**4.** Simplify $\frac{a(1-r^k)}{1-r}+ar^k$ to the next-case formula. [3 marks]

The numerator becomes $a[1-r^k+r^k(1-r)]=a(1-r^{k+1})$.

**Answer:** $\frac{a(1-r^{k+1})}{1-r}$.

**5.** Use an arithmetic sum to find $5+8+11+\cdots+62$. [2 marks]

$62=5+3(n-1)$ gives $n=20$; $\frac{20(5+62)}{2}=670$.

**Answer:** $670$.

**6.** Prove $2+6+\cdots+2\cdot 3^{n-1}=3^n-1$. [4 marks]

Base: $2=3-1$. Assume $S_k=3^k-1$.

Add $2\cdot 3^k$ to obtain $3^{k+1}-1$; conclude.

**Answer:** True for integers $n\geq 1$.