Introduction to proof by induction

6 questions | 16 marks

Equivalent correct methods earn the corresponding marks.

1. Check the base case for $1+2+\cdots+n=\frac{n(n+1)}{2}$. [2 marks]

The two sides agree for the first permitted integer.

Answer: At $n=1$, $1=\frac{1(2)}{2}$.

2. State the induction hypothesis for the same identity. [2 marks]

This assumes just the arbitrary k case.

Answer: Assume $1+2+\cdots+k=\frac{k(k+1)}{2}$ for arbitrary integer $k\geq 1$.

3. Complete the induction step for the same identity. [3 marks]

Factor $(k+1)$ from the two terms.

Answer: $\frac{k(k+1)}{2}+(k+1)=\frac{(k+1)(k+2)}{2}$.

4. A student checks $n=1,2,3$ and concludes an identity holds for every positive integer. Identify the missing argument. [2 marks]

Finite checks do not exclude later failure.

Answer: No implication for arbitrary k to $k+1$ has been established.

5. Prove $2+4+\cdots+2n=n(n+1)$ by induction. [4 marks]

Base: $2=1(2)$.

Assume $S_k=k(k+1)$. Add $2(k+1)$ to get $(k+1)(k+2)$; conclude.

Answer: True for all integers $n\geq 1$.

6. Why is an induction step alone insufficient for $1+2+\cdots+n=\frac{n(n+1)}{2}+5$? [3 marks]

A formal same-offset implication can hold without a true starting case.

Answer: The base fails: at $n=1$ the sides are 1 and 6.