Recurrence models

6 questions | 13 marks

Equivalent correct methods earn the corresponding marks.

1. $u_0 = 4, u_{n+1} = 3u_n + 2$. Find u_1, u_2, u_3. [2 marks]

Apply multiplication then addition at each step.

Answer: $14, 44, 134$.

2. A population starts at 200, keeps 90% yearly, then receives 50 newcomers. Find its rule and P_2. [2 marks]

$P_1 = 230$ and $P_2 = 257$.

Answer: $P_{n+1} = 0.9P_n + 50$, $P_0 = 200$; $P_2 = 257$.

3. Find the equilibrium and limiting value for $x_{n+1} = 0.75x_n + 8, x_0 = 0$. [2 marks]

$L = \frac{8}{1-0.75} = 32$; the deviation factor is 0.75.

Answer: Both equal 32.

4. For $u_{n+1} = 2u_n - 6, u_0 = 7$, find the equilibrium and long-term behaviour. [2 marks]

$u_n = 6 + 2^n$.

Answer: Equilibrium 6; the sequence increases without bound.

5. A machine adds 10 units then loses 20% of the new stock. Write a recurrence. [2 marks]

The new units experience the loss too.

Answer: $S_{n+1} = 0.8(S_n + 10) = 0.8S_n + 8$.

6. Find an explicit rule for $u_{n+1} = 0.5u_n + 3, u_0 = 10$. [3 marks]

The equilibrium is 6 and the initial deviation is 4.

Answer: $u_n = 6 + 4\left(\frac{1}{2}\right)^n$.