Infinite geometric sums and convergence

6 questions | 13 marks

Equivalent correct methods earn the corresponding marks.

1. Find $12+3+\frac{3}{4}+\cdots$ to infinity. [2 marks]

$\frac{12}{1-\frac{1}{4}}=16.$

Answer: $16.$

2. Find the infinite sum for $a=5, r=-0.6.$ [2 marks]

$\frac{5}{1+0.6}=3.125.$

Answer: $\frac{25}{8}=3.125.$

3. A geometric series starts at 8 and sums to 20. Find its ratio. [2 marks]

$\frac{8}{1-r}=20$ gives $r=\frac{3}{5}$.

Answer: $\frac{3}{5}$.

4. Use partial sums to explain why $1-1+1-1+\cdots$ does not converge. [2 marks]

They do not approach one value.

Answer: Partial sums alternate between 1 and 0.

5. A ball falls 2 m and rebounds to half its preceding height each time. Find the ideal total vertical distance. [3 marks]

The first fall is 2 m; later paired journeys total $2\left(1+\frac{1}{2}+\cdots\right)=4$ m.

Answer: $6\text{ m}.$

6. For $10+5+2.5+\cdots$, find the tail after four terms. [2 marks]

The fifth term is $10\left(\frac{1}{2}\right)^4=\frac{5}{8}$; its tail sums to $\frac{\frac{5}{8}}{\frac{1}{2}}=\frac{5}{4}$.

Answer: $1.25.$