Matrices and transformations - Practice test 5

Name: _____ Date: _____

10 questions | 63 marks | 70 minutes

Mixed applications: Connect resources, sensor readings, calibration, affine geometry and matrix equations.

No calculator needed.

1. A materials table is $A = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$, with rows for materials and columns for two products.

[6 marks]

(a) Find the resources needed for 2 of product 1 and 3 of product 2. [3 marks]

(b) The two material prices are 5 dollars and 2 dollars per unit. Calculate the total resource cost. [3 marks]

2. Let $A=\begin{pmatrix}1 & 0 & 2\\ 0 & 1 & -1\end{pmatrix}$ and $B=\begin{pmatrix}2 & 1\\ 3 & 0\\ -1 & 2\end{pmatrix}$. [6 marks]

(a) Find AB. [4 marks]

(b) State the order of BA and calculate its middle row. [2 marks]

3. Let $M=\begin{pmatrix}t & 1\\ 2 & 1\end{pmatrix}$ and consider $M(x,y)^T=(5,4)^T$. [6 marks]

(a) For t not equal to 2, solve for x and y in terms of t. [4 marks]

(b) Explain what happens at $t=2$. [2 marks]

4. First rotate 90° counterclockwise, then double only the x-coordinate. [6 marks]

(a) Find the combined matrix and describe its area effect. [3 marks]

(b) Find the images of the triangle $(0,0), (1,0), (0,2)$ and its image area. [3 marks]

5. Solve $AXB = C$ where $A = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 6 \\ 3 & 7 \end{pmatrix}$. [7 marks]

(a) Write a correct inverse expression for X. [2 marks]

(b) Calculate X and verify the product AXB. [5 marks]

6. A transformation is $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ followed by projection onto the x-axis. [6 marks]

(a) Find the combined matrix and explain its singularity. [3 marks]

(b) Describe every input giving image $(5, 0)$. [3 marks]

7. A point is rotated 180° about $c = (2, -1)$. [6 marks]

(a) Write the affine coordinate rule and find its fixed point. [3 marks]

(b) Can a 2×2 matrix alone describe this rotation in the original coordinates? Explain. [3 marks]

8. A map has matrix $M=\begin{pmatrix} k & 2 \\ 1 & k \end{pmatrix}$. An area-6 region has image area 12. [6 marks]

(a) Find every real value of k. [4 marks]

(b) Classify orientation for every possible k. [2 marks]

9. Two sensors report $u=x+y$ and $v=x-y$. Their outputs are $u=10$ and $v=2$. [6 marks]

(a) Use a matrix model to recover x and y. [3 marks]

(b) The second sensor fails, leaving only $u=10$. Describe all possible inputs and explain the loss of uniqueness. [3 marks]

10. A calibration sends $(1, 1)$ to $(4, 0)$ and $(1, -1)$ to $(0, 2)$. The map is linear. [8 marks]

(a) Find the calibration matrix. [4 marks]

(b) Find the input with calibrated image $(6, 1)$, and state the calibration area factor. [4 marks]