Matrices and transformations - Practice test 4

Name: _____ Date: _____

10 questions | 63 marks | 70 minutes

Reasoning and proof: Explain matrix claims with proofs, counterexamples and careful dimension checks.

No calculator needed.

1. Let $J=\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$. [6 marks]

(a) Apply J to $(1,0)$ and explain why J is not the identity. [3 marks]

(b) Find every vector left fixed by J. [3 marks]

2. Let $A=\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$, $B=\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}$. [6 marks]

(a) Calculate AB and explain what this disproves. [3 marks]

(b) Prove that if C is invertible and $CD=0$, then $D=0$. [3 marks]

3. Let $M=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ and $F=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$. [7 marks]

(a) Find the conditions on the entries for $MF=FM$. [4 marks]

(b) If M also commutes with $R=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$, what further condition is needed? [3 marks]

4. The matrix $P = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ projects onto the x-axis. [6 marks]

(a) Show that applying P twice has the same effect as once. [3 marks]

(b) Describe the image of the entire plane and give two different inputs with the same non-zero image. [3 marks]

5. Two linear maps have $\det A = -2$ and $\det B = 3$. [6 marks]

(a) A region has area 5. Find its area after A then B, and state the orientation change. [3 marks]

(b) Find the area multiplier of the inverse combined map. [3 marks]

6. Two matrices A and B are claimed to have AB of order 2×4 and BA of order 3×3. [6 marks]

(a) Show that no pair of matrices can meet both statements. [4 marks]

(b) Give compatible orders for A and B for which AB is 2×2 and BA is 3×3. [2 marks]

7. Let $S=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$. [6 marks]

(a) Find every point fixed by S and identify the transformation. [3 marks]

(b) Show that S is its own inverse and interpret the determinant. [3 marks]

8. Let $A = \begin{pmatrix} 2 & -1 \\ -4 & 2 \end{pmatrix}$. [6 marks]

(a) Describe the output line and all inputs mapped to zero. [3 marks]

(b) Find all inputs mapped to $(3, -6)$. [3 marks]

9. Let R be a 90° counterclockwise rotation and $t = (a, b)$ a translation vector. [7 marks]

(a) Express the two rules: translate by t then rotate; rotate then translate. [3 marks]

(b) For which t are the two rules equal for every input p? [4 marks]

10. Let $M = \begin{pmatrix} 1 & k \\ 0 & -1 \end{pmatrix}$. [7 marks]

(a) Show that M^2 is I for every real k. [3 marks]

(b) For which k is M an ordinary orthogonal reflection in the x-axis? Justify. [4 marks]