Matrices and transformations - Practice test 3

Name: _____ Date: _____

10 questions | 61 marks | 70 minutes

Inverses and design: Choose correct inverse operations and interpret constraints, areas and whole-item resource solutions.

No calculator needed.

1. Let $A = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$. [6 marks]

(a) Find its determinant and inverse. [3 marks]

(b) Solve $A(x, y)^T = (1, 5)^T$. [3 marks]

2. Let $A = \begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 2 \\ 1 & -1 \end{pmatrix}$. [6 marks]

(a) Solve $2X + A = 3B$. [3 marks]

(b) Show that your X has an inverse and calculate it. [3 marks]

3. A linear map sends $(1, 0)$ to $(0, -2)$ and $(0, 1)$ to $(3, 0)$. [6 marks]

(a) Find its matrix. Is it a pure rotation? [3 marks]

(b) Find the input whose image is $(9, 4)$. [3 marks]

4. First reflect in $y = x$, then rotate 90° counterclockwise, then dilate by 2 about the origin. [6 marks]

(a) Find the combined matrix and give one equivalent geometric description. [4 marks]

(b) Find the inverse matrix. [2 marks]

5. P is projection onto the x-axis. [6 marks]

(a) Describe all inputs with image $(3, 0)$. [3 marks]

(b) Suppose the original point is known to lie on $y = 2x + 1$. Recover it. Does this make P globally invertible? [3 marks]

6. Let $M = \begin{pmatrix} 1 & k \\ k & 1 \end{pmatrix}$. [7 marks]

(a) Find the values of k for which M is singular and describe the output line for each. [4 marks]

(b) Find k if the image of a unit square has area 3. [3 marks]

7. Consider $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$. [6 marks]

(a) Explain why A preserves area but not every length. [3 marks]

(b) State the area factors of A^2 and A^{-1}. [3 marks]

8. A coordinate rule is $q = Rp + (4, 2)$, where R rotates 90° counterclockwise. [6 marks]

(a) Find p when $q = (5, -1)$. [3 marks]

(b) Find the image of the origin and explain why the inverse must include a translation. [3 marks]

9. Let $A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. [6 marks]

(a) Calculate AB and its inverse. [4 marks]

(b) Is $(AB)^{-1} = A^{-1}B^{-1}$ here? Explain. [2 marks]

10. A workshop uses 2 units of timber and 1 of metal per small item, and 1 unit of timber and 3 of metal per large item. [6 marks]

(a) There are exactly 11 timber units and 13 metal units to use. Find the number of each item. [3 marks]

(b) If timber changes to 12 units while metal stays 13, can both stocks be used exactly with whole items? [3 marks]