Matrices and transformations - Practice test 2

Name: _____ Date: _____

10 questions | 62 marks | 70 minutes

Geometry and matrix models: Use matrices for cost changes, geometric mappings, projection, rotations and output constraints.

No calculator needed.

1. A store records notebook, folder and pen quantities in $N=\begin{pmatrix}6 & 4 & 3\\ 2 & 5 & 7\end{pmatrix}$, one row per day. Prices are $p=(4,3,2)^T$ dollars. [6 marks]

(a) Calculate Np and interpret it. [3 marks]

(b) Prices change by $(1,0,-0.5)^T$ dollars. Find the change in each daily total. [3 marks]

2. A linear map sends $(1, 1)$ to $(3, 1)$ and $(1, -1)$ to $(1, 3)$. [7 marks]

(a) Find the images of the two coordinate unit vectors and hence the matrix. [4 marks]

(b) Find the image of $(2, 3)$ and the area multiplier. [3 marks]

3. Let $F = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $D = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$. [6 marks]

(a) Calculate FD and DF. [4 marks]

(b) Apply both products to $(2, 1)$ and explain the different geometric orders. [2 marks]

4. A point is rotated 60° counterclockwise about the origin. [6 marks]

(a) Write the exact rotation matrix. [3 marks]

(b) Find the image of $(2, 0)$ and give the inverse transformation. [3 marks]

5. Let P_x be projection onto the x-axis and R a 90° counterclockwise rotation. [6 marks]

(a) Find P_xR and apply it to $(3, 4)$. [3 marks]

(b) Find RP_x and apply it to the same point. Is either product invertible? [3 marks]

6. Let $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and solve $XA = \begin{pmatrix} 2 & 5 \\ -1 & 1 \end{pmatrix}$. [6 marks]

(a) Find X. [3 marks]

(b) Calculate AX and explain why it need not equal XA. [3 marks]

7. A map has matrix $M = \begin{pmatrix} k & 1 \\ 2 & k \end{pmatrix}$. A region of area 3 becomes a region of area 21. [6 marks]

(a) Find every possible real k. [4 marks]

(b) For each permitted k, state whether orientation is preserved. [2 marks]

8. Let $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$. [6 marks]

(a) Find every input mapped to $(5, 10)$. [3 marks]

(b) Explain why no input maps to $(5, 11)$. [3 marks]

9. The unit square is transformed by $A = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$. [7 marks]

(a) Give its four image vertices in cyclic order and find its area. [4 marks]

(b) Find the acute angle between the two edges meeting at the origin. [3 marks]

10. A rotation through 90° counterclockwise is about the centre $c = (1, -2)$. [6 marks]

(a) Find the image of $p = (4, -1)$. [3 marks]

(b) Find the input whose image is $(2, 4)$. [3 marks]