Matrices and transformations - Practice test 1

Name: _____ Date: _____

10 questions | 61 marks | 70 minutes

Foundations and fluency: Build from notation and arithmetic to transformations, singularity and an affine model.

No calculator needed.

1. Let $A=\begin{pmatrix} 1 & 2 & 0 \\ -1 & 3 & 2 \end{pmatrix}$ and $B=\begin{pmatrix} 2 & 1 \\ 0 & -1 \\ 1 & 4 \end{pmatrix}$. [6 marks]

(a) State the order of A and the entry a_{23}. [2 marks]

(b) Calculate AB. [4 marks]

2. Let $C=\begin{pmatrix}2 & -1\\ 1 & 3\end{pmatrix}$, $D=\begin{pmatrix}0 & 4\\ -2 & 1\end{pmatrix}$. [6 marks]

(a) Calculate $2C-D$. [3 marks]

(b) Solve $3X+C=D$. [3 marks]

3. Let $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$. [6 marks]

(a) Calculate AB and BA. [4 marks]

(b) Give one input vector for which the two combined transformations have different images. [2 marks]

4. Let $M = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$. [6 marks]

(a) Find the determinant and inverse of M. [3 marks]

(b) Use the inverse to solve $M(x, y)^T = (8, 7)^T$. [3 marks]

5. Consider $A = \begin{pmatrix} k & 2 \\ 3 & 6 \end{pmatrix}$. [7 marks]

(a) Find k for which A is singular. [3 marks]

(b) For $k = 1$, compare the solutions of $x + 2y = 4,\ 3x + 6y = 12$ with those of $x + 2y = 4,\ 3x + 6y = 13$. [4 marks]

6. A linear map sends e_1 to $(2, 1)$ and e_2 to $(-1, 2)$. [6 marks]

(a) Write its matrix. [2 marks]

(b) Find the image of $(3, -2)$ and its area scale factor. [4 marks]

7. A point is reflected in the x-axis, then rotated 90° counterclockwise about the origin. [6 marks]

(a) Find the combined matrix. [3 marks]

(b) Find the image of $(2, -3)$ and identify the single equivalent transformation. [3 marks]

8. Let P be orthogonal projection onto the y-axis. [6 marks]

(a) Write P and calculate its determinant. [3 marks]

(b) Describe every input mapped to the origin, and explain why P has no inverse. [3 marks]

9. Triangle OAB has $O=(0,0)$, $A=(4,0)$ and $B=(0,3)$. It is transformed by $T=\begin{pmatrix}2 & 1\\ 0 & -1\end{pmatrix}$. [6 marks]

(a) Find its original area and the images of A and B. [3 marks]

(b) Find the image area and say whether orientation is preserved. [3 marks]

10. A camera coordinate rule is $q=\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}p+(3,\,-1)$. [6 marks]

(a) Find p when $q=(13,3)$. [3 marks]

(b) Can one 2×2 matrix acting alone represent this entire rule? Explain. [3 marks]