Determinant and area scale

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Find the area factor of $\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$. [2 marks]

The determinant is 6.

Answer: 6.

2. An area-5 region is mapped by $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Find its image area and orientation change. [2 marks]

The determinant is -2.

The area factor is 2.

Answer: Area 10; orientation reverses.

3. Triangle $(0,0), (2,0), (0,3)$ is dilated by factor 4. Find its image area. [2 marks]

Its original area is $\frac{1}{2}(2)(3) = 3$.

The determinant of $4I$ is 16, so the image area is 48.

Answer: 48 square units.

4. If $\det A = -5$, state the determinant and area factor of A^{-1}. [2 marks]

The inverse determinant is the reciprocal.

Take the absolute value for area.

Answer: Determinant $-\frac{1}{5}$; area factor $\frac{1}{5}$.

5. Find the area effect of $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$. [2 marks]

The determinant is zero, so the map is singular.

Answer: Every image region has area zero.

6. The diagonal matrix $\begin{pmatrix} k & 0 \\ 0 & 3 \end{pmatrix}$ sends an area-4 region to area 24. Find all real k. [2 marks]

The area factor is 6, so $|3k| = 6$.

Thus the magnitude of k is 2.

Answer: $k = 2$ or $k = -2$.