Solving matrix equations and systems

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Write $3x - y = 4$, $2x + 5y = 1$ as one matrix equation. [2 marks]

Keep x first and y second in both rows.

Answer: $\begin{pmatrix} 3 & -1 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$.

2. Solve $x + y = 5$, $x - y = 1$. [2 marks]

Add the equations to get $2x = 6$.

Then $y = 5 - 3 = 2$; the matrix method gives the same result.

Answer: $x = 3$, $y = 2$.

3. Solve $2x + 3y = 1$, $x + 2y = 2$. [2 marks]

The determinant is 1.

The inverse $\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$ applied to $(1, 2)^T$ gives $(-4, 3)^T$.

Answer: $x = -4$, $y = 3$.

4. Solve $\begin{pmatrix}2 & 0\\ 0 & 3\end{pmatrix}X=\begin{pmatrix}8\\ 9\end{pmatrix}$. [2 marks]

Undo the two diagonal scales.

Answer: $X=\begin{pmatrix}4\\ 3\end{pmatrix}$.

5. Solve $X\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}=\begin{pmatrix}3 & 7\\ 1 & 4\end{pmatrix}$. [2 marks]

Multiply the right-hand matrix on the right by $\begin{pmatrix}1 & -2\\ 0 & 1\end{pmatrix}$.

Answer: $X=\begin{pmatrix}3 & 1\\ 1 & 2\end{pmatrix}$.

6. A ticket model gives $2a+b=17$ and $a+2b=19$. Find a and b and check both equations. [2 marks]

Eliminate to obtain $3a=15$.

Then $b=17-10=7$; the second equation gives $5+14=19$.

Answer: $a=5,\ b=7$.