Singular matrices

6 questions | 12 marks

Equivalent correct methods earn the corresponding marks.

1. Is $\begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix}$ singular? [2 marks]

The determinant is $6-6=0$.

Answer: Yes.

2. Find k so that $\begin{pmatrix} 2 & k \\ 4 & 6 \end{pmatrix}$ is singular. [2 marks]

Set $12-4k=0$.

Answer: $k=3$.

3. Solve $x+y=4$, $2x+2y=8$ using a parameter. [2 marks]

The second equation repeats the first.

Choose $y=t$ and obtain $x=4-t$.

Answer: $(x, y)=(4-t, t)$ for any real t.

4. Explain why $x+y=4$, $2x+2y=9$ is inconsistent. [2 marks]

No pair can satisfy both statements.

Answer: Doubling the first equation gives 8, not 9.

5. Find a non-zero vector sent to zero by $\begin{pmatrix} 1 & -2 \\ 2 & -4 \end{pmatrix}$. [2 marks]

$2-2(1)=0$ and $2(2)-4(1)=0$.

Answer: For example $(2, 1)$.

6. The matrix $\begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix}$ acts on the plane. Describe all its possible outputs. [2 marks]

The output is $(3x, 0)$.

Every real x-coordinate is possible, but the y-coordinate is always zero.

Answer: All points on the x-axis.