Vectors in two dimensions - Practice test 5

Name: _____ Date: _____

10 questions | 61 marks | 75 minutes

A more demanding synthesis paper combining unknowns, modelling, projections and general proofs.

A scientific calculator may be used.

1. Let $\mathbf{a} = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{b} = -\mathbf{i} + 3\mathbf{j}$. [4 marks]

(a) Find $3\mathbf{a} + 2\mathbf{b}$ in component form. [2 marks]

(b) Find a unit vector in this resultant's direction. [2 marks]

2. $A = (k, 2)$, $B = (5, m)$. The displacement BA is $(-3, 4)$. [4 marks]

(a) Find k and m. [2 marks]

(b) Find the exact length of AB and a unit vector from A to B. [2 marks]

3. A non-zero vector v is parallel to $(3, 4)$, has magnitude 20 and has a negative x-component. [6 marks]

(a) Find v. [3 marks]

(b) Find its direction from the positive x-axis to 0.1°. [3 marks]

4. Non-zero vectors a and b satisfy $|a| = 2$, $|b| = 3$ and $a \cdot b = 3$. [6 marks]

(a) Find their included angle. [2 marks]

(b) Find $|2\mathbf{a} - \mathbf{b}|$ and the scalar projection of a onto b. [4 marks]

5. $A=(-1,2)$, $B=(3,4)$, $C=(6,-2)$. D completes the parallelogram $ABCD$ in that order. [7 marks]

(a) Find D. [3 marks]

(b) Prove that the parallelogram is a rectangle and find its area. [4 marks]

6. A vector u is perpendicular to $(1,2)$, and its vector projection onto the positive x-axis is $(-6,0)$. [5 marks]

(a) Find u. [3 marks]

(b) Find its scalar projection onto $(0,-1)$. [2 marks]

7. Two adjustable forces are $m(3, 4)$ N and $n(2, -2)$ N, where m,n are positive. Their resultant must be $(14, 0)$ N. [7 marks]

(a) Find m and n. [4 marks]

(b) Compare the sum of the force magnitudes with the resultant magnitude. [3 marks]

8. Let $u = (1, t)$ and $v = (t, 1)$, where t is real. [7 marks]

(a) Find all t making u and v parallel, and give the corresponding angles. [4 marks]

(b) Find all t making u and v perpendicular. [3 marks]

9. A line through O has direction $v = (3, 4)$. P has position $p = (-4, 3)$. A point Q on the line has position $t\mathbf{v}$. [7 marks]

(a) Show that $|\overrightarrow{PQ}|^2 = 25 + 25t^2$. [4 marks]

(b) Hence find the closest point on the line to P and the shortest distance, without calculus. [3 marks]

10. In a quadrilateral $ABCD$, M,N,P,Q are the midpoints of AB,BC,CD,DA respectively. Assume the midpoint quadrilateral is non-degenerate. [8 marks]

(a) Use position vectors to prove that $MNPQ$ is a parallelogram. [4 marks]

(b) Prove that perpendicular diagonals AC and BD make $MNPQ$ a rectangle. [4 marks]