Vectors in two dimensions - Practice test 4

Name: _____ Date: _____

10 questions | 54 marks | 70 minutes

Apply vectors to routes, forces and geometric decisions; distinguish a calculation from its interpretation.

A scientific calculator may be used.

1. A rescue vehicle's displacement is 3 km west and 4 km north. [5 marks]

(a) Write its Cartesian displacement and find the distance from its starting point. [2 marks]

(b) Find its bearing, measured clockwise from north, to 0.1°. [3 marks]

2. $A = (2, -1)$, $B = (6, 5)$, $C = (-3, 4)$. [4 marks]

(a) Find D such that $CD = AB$. [2 marks]

(b) Find E such that $CE = BA$. [2 marks]

3. A student claims that $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$ for every pair of vectors. [5 marks]

(a) Give a counterexample using $a = (3, 0)$ and $b = (0, 4)$. [3 marks]

(b) State a condition on non-zero a and b that makes the claimed equality true. [2 marks]

4. $A = (2, -1)$, $B = (8, 2)$ and $\mathbf{p} = \mathbf{a} - \frac{1}{2}(\mathbf{b} - \mathbf{a})$. [6 marks]

(a) Find P and explain where it lies. [3 marks]

(b) Find the ratio $AP : PB$ of lengths. [3 marks]

5. For this question, work done by a constant force is defined by $W = \mathbf{F} \cdot \mathbf{d}$. Force is measured in newtons and displacement in metres. Let $F = (4, 3)$ and $d = (2, -1)$. [5 marks]

(a) Find W and give its unit. [2 marks]

(b) Find the signed component of F along the direction of d. [3 marks]

6. A vector u satisfies $\mathbf{u} \cdot (1, 1) = 5$ and $\mathbf{u} \cdot (2, -1) = 1$. [5 marks]

(a) Find u. [3 marks]

(b) Find its magnitude and the unit vector opposite to it. [2 marks]

7. Let $a=(2,1)$, $b=(-1,2)$. [5 marks]

(a) Show that $a+b$ and $a-b$ are perpendicular. [2 marks]

(b) Find the angle between a and $a+b$. [3 marks]

8. $A=(1,2)$, $B=(5,4)$ and $P=(4,5)$. H is the perpendicular foot from P to the infinite line AB. [7 marks]

(a) Find H by projecting AP onto AB. [4 marks]

(b) Find PH and explain whether H is on segment AB. [3 marks]

9. A vector has magnitude 10. Its signed scalar projection onto the positive x-axis is 6. [5 marks]

(a) Find every possible vector. [3 marks]

(b) Find the magnitude of its perpendicular component. [2 marks]

10. A, B and C are the vertices of a non-degenerate triangle, with position vectors a, b and c. Define G by $\mathbf{g} = \frac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}$. Let M be the midpoint of BC. [7 marks]

(a) Show $\overrightarrow{AG} = \frac{2}{3}\overrightarrow{AM}$. [4 marks]

(b) State the ratio $AG : GM$ and explain why G is on the median AM. [3 marks]