Vectors in two dimensions - Practice test 4

10 questions | 54 marks | 70 minutes

Apply vectors to routes, forces and geometric decisions; distinguish a calculation from its interpretation.

Equivalent correct methods earn the corresponding marks.

1. A rescue vehicle's displacement is 3 km west and 4 km north. [5 marks]

(a) Write its Cartesian displacement and find the distance from its starting point. [2 marks]

East is positive x and north is positive y.

Use Pythagoras for the magnitude.

Answer: $(-3, 4)$ km; 5 km.

(b) Find its bearing, measured clockwise from north, to 0.1°. [3 marks]

It is $\tan^{-1}\left(\frac{3}{4}\right) = 36.869\ldots^\circ$ west of north.

$\text{Bearing} = 360^\circ - 36.869\ldots^\circ$.

Answer: 323.1°.

2. $A = (2, -1)$, $B = (6, 5)$, $C = (-3, 4)$. [4 marks]

(a) Find D such that $CD = AB$. [2 marks]

$AB = (4, 6)$; add that displacement to C.

Answer: $D = (1, 10)$.

(b) Find E such that $CE = BA$. [2 marks]

Reverse AB to $(-4, -6)$, then add it to C.

Answer: $E = (-7, -2)$.

3. A student claims that $|\mathbf{a}+\mathbf{b}|=|\mathbf{a}|+|\mathbf{b}|$ for every pair of vectors. [5 marks]

(a) Give a counterexample using $a=(3,0)$ and $b=(0,4)$. [3 marks]

The sum is $(3,4)$ with length five.

The individual lengths are three and four.

Answer: The left side is 5, but the right side is 7.

(b) State a condition on non-zero a and b that makes the claimed equality true. [2 marks]

Head-to-tail addition then forms one straight segment without a reversal.

Answer: They point in the same direction.

4. $A=(2,\ -1)$, $B=(8,2)$ and $\mathbf{p}=\mathbf{a}-\frac{1}{2}(\mathbf{b}-\mathbf{a})$. [6 marks]

(a) Find P and explain where it lies. [3 marks]

$AB=(6,3)$; subtract half of it from A.

The negative parameter means the opposite direction.

Answer: $P=(-1,\ -\frac{5}{2})$, beyond A away from B.

(b) Find the ratio $AP:PB$ of lengths. [3 marks]

AP is half the length of AB, while PB is one and a half times AB.

Answer: $1:3$.

5. For this question, work done by a constant force is defined by $W=\mathbf{F}\cdot\mathbf{d}$. Force is measured in newtons and displacement in metres. Let $F=(4,3)$ and $d=(2,\ -1)$. [5 marks]

(a) Find W and give its unit. [2 marks]

$4(2)+3(-1)=5$; newton metres are joules.

Answer: 5 J.

(b) Find the signed component of F along the direction of d. [3 marks]

Divide the dot product five by $|d|=\sqrt{5}$.

Answer: $\sqrt{5}$ N.

6. A vector u satisfies $\mathbf{u} \cdot (1, 1) = 5$ and $\mathbf{u} \cdot (2, -1) = 1$. [5 marks]

(a) Find u. [3 marks]

Write $u = (x, y)$: $x + y = 5$ and $2x - y = 1$.

Adding gives $3x = 6$, then $y = 3$.

Answer: $(2, 3)$.

(b) Find its magnitude and the unit vector opposite to it. [2 marks]

Use Pythagoras, then negate and divide by the length.

Answer: $\sqrt{13}$; $(-\frac{2}{\sqrt{13}}, -\frac{3}{\sqrt{13}})$.

7. Let $a = (2, 1)$, $b = (-1, 2)$. [5 marks]

(a) Show that $a + b$ and $a - b$ are perpendicular. [2 marks]

Calculate both sum and difference.

Answer: Their dot product is $(1, 3) \cdot (3, -1) = 0$.

(b) Find the angle between a and $a + b$. [3 marks]

Their dot product is five; their lengths are $\sqrt{5}$ and $\sqrt{10}$.

The cosine is $\frac{1}{\sqrt{2}}$.

Answer: 45°.

8. $A = (1, 2)$, $B = (5, 4)$ and $P = (4, 5)$. H is the perpendicular foot from P to the infinite line AB. [7 marks]

(a) Find H by projecting AP onto AB. [4 marks]

$AP = (3, 3)$, $AB = (4, 2)$; projection coefficient is $\frac{18}{20} = \frac{9}{10}$.

Add $(\frac{18}{5}, \frac{9}{5})$ to A.

Answer: $H = (\frac{23}{5}, \frac{19}{5})$.

(b) Find PH and explain whether H is on segment AB. [3 marks]

$P - H = (-\frac{3}{5}, \frac{6}{5})$, whose magnitude is $\frac{3}{\sqrt{5}}$.

The parameter $\frac{9}{10}$ lies between zero and one.

Answer: $PH = \frac{3}{\sqrt{5}}$; H is on the segment.

9. A vector has magnitude 10. Its signed scalar projection onto the positive x-axis is 6. [5 marks]

(a) Find every possible vector. [3 marks]

The x-component is six; $36 + y^2 = 100$.

Thus $y = \pm 8$.

Answer: $(6, 8)$ or $(6, -8)$.

(b) Find the magnitude of its perpendicular component. [2 marks]

The perpendicular component is $(0, \pm 8)$, with length eight.

Answer: 8.

10. A, B and C are the vertices of a non-degenerate triangle, with position vectors a, b and c. Define G by $\mathbf{g} = \frac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}$. Let M be the midpoint of BC. [7 marks]

(a) Show $\overrightarrow{AG} = \frac{2}{3}\overrightarrow{AM}$. [4 marks]

Subtract a from g.

Use $OM = \frac{b+c}{2}$ to calculate AM.

Answer: Both sides equal $\frac{\mathbf{b}+\mathbf{c}-2\mathbf{a}}{3}$.

(b) State the ratio $AG : GM$ and explain why G is on the median AM. [3 marks]

The remaining third of AM is GM.

The vector scalar multiple proves collinearity.

Answer: $2 : 1$; the positive parameter $\frac{2}{3}$ places G on AM.