Vectors in two dimensions - Practice test 3

Name: _____ Date: _____

10 questions | 52 marks | 70 minutes

Connect exact values, projections and geometric structure, with a short proof to finish.

A scientific calculator may be used.

1. Let $\mathbf{v} = (4\sqrt{3}, -4)$. [4 marks]

(a) Find its magnitude and direction from the positive x-axis. [2 marks]

(b) Find the unit vector in the same direction. [2 marks]

2. In an original coordinate system $A = (-3, 2)$ and $B = (1, 5)$. A new origin is placed at old coordinates $(2, -1)$, with axes pointing in the same directions. [5 marks]

(a) Give the new coordinates of A and B. [2 marks]

(b) Calculate AB in both systems and explain the agreement. [3 marks]

3. A triangular route starts with displacement $a = (2, 1)$ km, then $b = (-1, 4)$ km, then c back to the starting point. [5 marks]

(a) Find c. [2 marks]

(b) Find the exact total distance travelled. [3 marks]

4. Let $u = (2, 1)$, $v = (t, 3)$ and $w = (4, s)$. [4 marks]

(a) Find t if v is parallel to u. [2 marks]

(b) Find s if w is perpendicular to u. [2 marks]

5. $A=(1,0)$, $B=(5,2)$, $C=(3,6)$. [6 marks]

(a) Find angle BAC exactly. [3 marks]

(b) Use a second vector calculation to classify the triangle. [3 marks]

6. A force $u=(-4,1)$ N is resolved along the direction $v=(3,0)$. [4 marks]

(a) Find its signed scalar projection and vector projection along v. [2 marks]

(b) Find the perpendicular component. [2 marks]

7. A line passes through O and has direction $v = (2, 1)$. P has position $u = (5, 3)$. [6 marks]

(a) Find the position of the perpendicular foot H on the line. [3 marks]

(b) Find the exact shortest distance from P to the line. [3 marks]

8. The angle between $(a, 1)$ and $(1, 1)$ is 45°. [6 marks]

(a) Form an equation involving a using the dot product. [3 marks]

(b) Find a and check the original angle condition. [3 marks]

9. A parallelogram has adjacent sides $a = (5, 0)$ and $b = (3, 4)$. [6 marks]

(a) Show that it is a rhombus and that its diagonals are perpendicular. [3 marks]

(b) Find its area using half the product of the diagonal lengths. [3 marks]

10. Non-zero vectors a and b satisfy $|a| = |b| = L$ and $a \cdot b = \frac{L^2}{2}$. O, A and B have position vectors 0, a and b. [6 marks]

(a) Prove that OAB is equilateral. [3 marks]

(b) If M is the midpoint of AB, find the length OM. [3 marks]