Vectors in two dimensions - Practice test 1

Name: _____ Date: _____

10 questions | 46 marks | 65 minutes

Build from components and direction to projections and a geometric proof.

A scientific calculator may be used.

1. Let $\mathbf{a} = (3, 4)$ and $\mathbf{b} = (-2, 1)$. [4 marks]

(a) Find $2\mathbf{a} - \mathbf{b}$. [2 marks]

(b) Find $|\mathbf{a}|$ and a unit vector in its direction. [2 marks]

2. $A = (1, -2)$, $B = (7, 6)$ and $C = (-2, 5)$. [4 marks]

(a) Find AB and its length. [2 marks]

(b) Find D if $CD = BA$. [2 marks]

3. The vector $\mathbf{u} = (-5, 5)$ describes a displacement in kilometres. [4 marks]

(a) Find its exact magnitude and its direction anticlockwise from the positive x-axis. [2 marks]

(b) Find the displacement directly back to the start. [2 marks]

4. $A = (2, 1)$, $B = (10, 5)$. [4 marks]

(a) Find the point P for $AP : PB = 3 : 1$. [2 marks]

(b) Find Q beyond B such that $\overrightarrow{AQ} = \frac{3}{2}\overrightarrow{AB}$. [2 marks]

5. Let $\mathbf{p} = (2, 3)$ and $\mathbf{q} = (8, k)$. [4 marks]

(a) Find k if p and q are parallel. [2 marks]

(b) Using your k, state whether p and q have the same or opposite directions. [2 marks]

6. Let $\mathbf{u} = (1, 2)$ and $\mathbf{v} = (2, -1)$. [4 marks]

(a) Calculate $\mathbf{u} \cdot \mathbf{v}$ and state the angle between them. [2 marks]

(b) Find $|\mathbf{u} + \mathbf{v}|$ and check it against $|\mathbf{u}|^2 + |\mathbf{v}|^2$. [2 marks]

7. Let $\mathbf{a} = (4, 2)$ and $\mathbf{b} = (1, 1)$. [5 marks]

(a) Find the scalar projection of a onto b. [2 marks]

(b) Find the vector projection and the perpendicular remainder. [3 marks]

8. A force vector is $\mathbf{F} = (6, -3)$ N. The unit direction along a rail is $\mathbf{e} = (\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}})$. [5 marks]

(a) Find the signed component of F along the rail. [2 marks]

(b) Find the component vector along the rail. [3 marks]

9. $A=(0,0)$, $B=(4,1)$, $C=(3,5)$. [6 marks]

(a) Show that angle ABC is a right angle. [3 marks]

(b) Find the exact area of triangle ABC. [3 marks]

10. In triangle OAB, $OA=a$ and $OB=b$. M and N are the midpoints of OA and OB. [6 marks]

(a) Express MN in terms of a and b. [3 marks]

(b) Prove that MN is parallel to AB and half its length. [3 marks]