Vectors in two dimensions - Practice test 1

10 questions | 46 marks | 65 minutes

Build from components and direction to projections and a geometric proof.

Equivalent correct methods earn the corresponding marks.

1. Let $\mathbf{a} = (3, 4)$ and $\mathbf{b} = (-2, 1)$. [4 marks]

(a) Find $2\mathbf{a} - \mathbf{b}$. [2 marks]

$2\mathbf{a} = (6, 8)$; subtract $(-2, 1)$.

Answer: $(8, 7)$.

(b) Find $|\mathbf{a}|$ and a unit vector in its direction. [2 marks]

$|\mathbf{a}| = \sqrt{9 + 16} = 5$; divide by five.

Answer: 5 and $(\frac{3}{5}, \frac{4}{5})$.

2. $A = (1, -2)$, $B = (7, 6)$ and $C = (-2, 5)$. [4 marks]

(a) Find AB and its length. [2 marks]

$AB = (7 - 1, 6 + 2) = (6, 8)$; length 10.

Answer: $(6, 8)$; 10.

(b) Find D if $CD = BA$. [2 marks]

$BA = (-6, -8)$; add to C.

Answer: $D = (-8, -3)$.

3. The vector $\mathbf{u} = (-5, 5)$ describes a displacement in kilometres. [4 marks]

(a) Find its exact magnitude and its direction anticlockwise from the positive x-axis. [2 marks]

Magnitude $\sqrt{25+25} = 5\sqrt{2}$; the direction is quadrant II.

Answer: $5\sqrt{2}$ km and $135°$.

(b) Find the displacement directly back to the start. [2 marks]

Negate the original displacement.

Answer: $(5, -5)$ km.

4. $A = (2, 1)$, $B = (10, 5)$. [4 marks]

(a) Find the point P for $AP : PB = 3 : 1$. [2 marks]

$AB = (8, 4)$; add three quarters of it to A.

Answer: $(8, 4)$.

(b) Find Q beyond B such that $\overrightarrow{AQ} = \frac{3}{2}\overrightarrow{AB}$. [2 marks]

$A + \frac{3}{2}(8, 4) = (2, 1) + (12, 6)$.

Answer: $(14, 7)$.

5. Let $\mathbf{p} = (2, 3)$ and $\mathbf{q} = (8, k)$. [4 marks]

(a) Find k if p and q are parallel. [2 marks]

The first component is multiplied by four, so the second is too.

Answer: $k = 12$.

(b) Using your k, state whether p and q have the same or opposite directions. [2 marks]

$\mathbf{q} = 4\mathbf{p}$, a positive multiple.

Answer: Same direction.

6. Let $\mathbf{u} = (1, 2)$ and $\mathbf{v} = (2, -1)$. [4 marks]

(a) Calculate $\mathbf{u} \cdot \mathbf{v}$ and state the angle between them. [2 marks]

$1(2) + 2(-1) = 0$; both vectors are non-zero.

Answer: 0 and 90°.

(b) Find $|\mathbf{u} + \mathbf{v}|$ and check it against $|\mathbf{u}|^2 + |\mathbf{v}|^2$. [2 marks]

The sum is $(3, 1)$ with squared magnitude ten.

Answer: $\sqrt{10}$; its square is $10 = 5 + 5$.

7. Let $\mathbf{a} = (4, 2)$ and $\mathbf{b} = (1, 1)$. [5 marks]

(a) Find the scalar projection of a onto b. [2 marks]

$\mathbf{a} \cdot \mathbf{b} = 6$; divide by $\sqrt{2}$.

Answer: $3\sqrt{2}$.

(b) Find the vector projection and the perpendicular remainder. [3 marks]

Projection coefficient $\frac{6}{2} = 3$; subtract $(3, 3)$ from a.

Answer: Projection $(3, 3)$; remainder $(1, -1)$.

8. A force vector is $\mathbf{F} = (6, -3)$ N. The unit direction along a rail is $\mathbf{e} = (\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}})$. [5 marks]

(a) Find the signed component of F along the rail. [2 marks]

Dot F with the supplied unit vector: $\frac{12 - 3}{\sqrt{5}}$.

Answer: $\frac{9}{\sqrt{5}}$ N.

(b) Find the component vector along the rail. [3 marks]

Multiply $\frac{9}{\sqrt{5}}$ by e.

Answer: $(\frac{18}{5}, \frac{9}{5})$ N.

9. $A=(0,0)$, $B=(4,1)$, $C=(3,5)$. [6 marks]

(a) Show that angle ABC is a right angle. [3 marks]

Both vectors start at B and are non-zero.

Answer: $BA \cdot BC = (-4,\ -1) \cdot (-1, 4) = 0$.

(b) Find the exact area of triangle ABC. [3 marks]

The perpendicular sides each have length $\sqrt{17}$.

$\text{Area} = \frac{1}{2}\sqrt{17}\sqrt{17} = \frac{17}{2}$.

Answer: $\frac{17}{2}$ square units.

10. In triangle OAB, $OA = a$ and $OB = b$. M and N are the midpoints of OA and OB. [6 marks]

(a) Express MN in terms of a and b. [3 marks]

$ON = \frac{1}{2}b$ and $OM = \frac{1}{2}a$; subtract.

Answer: $\frac{1}{2}(\mathbf{b} - \mathbf{a})$.

(b) Prove that MN is parallel to AB and half its length. [3 marks]

$AB = b - a$; the positive factor $\frac{1}{2}$ preserves direction and halves magnitude.

Answer: $\overrightarrow{MN} = \frac{1}{2}\overrightarrow{AB}$, proving both statements.