Scalar/vector projections

6 questions | 16 marks

Equivalent correct methods earn the corresponding marks.

1. Find the scalar projection of $(3, 4)$ onto $(1, 0)$. [2 marks]

The direction has length one and the dot product is three.

Answer: 3.

2. Find the vector projection of $(2, 5)$ onto $(0, 2)$. [2 marks]

$\text{Coefficient} = (\frac{10}{4}) = \frac{5}{2}$.

Multiply by $(0, 2)$.

Answer: $(0, 5)$.

3. Project $(5, 1)$ onto $(1, 1)$. [2 marks]

The dot product is six and squared direction length is two.

Answer: $(3, 3)$.

4. Resolve $(4, 2)$ into components parallel and perpendicular to $(1, 1)$. [3 marks]

Calculate the projection, then subtract it.

Check $(1, -1) \cdot (1, 1) = 0$.

Answer: Parallel $(3, 3)$; perpendicular $(1, -1)$.

5. Find the scalar projection of $(-2, 3)$ onto $(2, 0)$ and the length of the projected vector. [3 marks]

The dot product is -4, divided by direction length 2.

Take the absolute value only for the length.

Answer: Scalar -2; length 2.

6. Show that replacing v by $2\mathbf{v}$ does not change $\text{proj}_{\mathbf{v}}\,\mathbf{u}$. [4 marks]

The numerator doubles, the denominator quadruples and the final direction vector doubles.

The factors cancel.

Answer: Both give $\frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}}\mathbf{v}$.